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Integration of dimensional quality and locator reliability in design and evaluation of multi-station body-in-white assembly processes

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Research efforts have been made to develop a Quality and Reliability chain (QR chain) model to integrate manufacturing system component reliability and product quality in multi-station manufacturing processes. Based on a previously developed state space model, which captures the variation propagation throughout all stations, a general QR chain model is developed for Body-In-White (BIW) assembly processes. The effectiveness of the QR chain modeling strategy is demonstrated by thoroughly studying the relationship between locator performance and product quality in assembly processes. Based on the analytical result of system reliability obtained from the QR chain model, optimal locator wear rate assignment is further investigated. A case study is conducted to demonstrate the effectiveness and potential usage of the QR chain model for the BIW assembly system design evaluation and optimization.

1. Introduction

An automotive body without doors, hood, fenders and trunk lid is called a "Body-In-White" (BIW). In a BIW assembly line, depending on the complexity of the product, there are typically 80 to 130 assembly stations that assemble 150 to 250 sheet metal parts. System reliability of the BIW assembly process is one of the key factors affecting the productivity and product quality.

In general, system failure of a BIW assembly process can occur either due to catastrophic tooling failure or by an unsatisfactory product quality. A tooling failure such as a broken and loose locating pin can directly lead to an immediate halt in the production process. A nonconforming product quality, for example a large product variation, is an indication that the process is unable to produce products with the specified quality.

Also, data collected in real world situations have shown that significant interactions occur between the tool reliability and the product quality and that these interactions propagate through each station of a BIW assembly process. For example, previous researches indicated that 72% of the root causes of dimensional errors of a BIW are due to locating tool malfunction (Ceglarek and Shi, 1995), which indicates the significant effects of locating tool reliability on the dimensional product quality. On the other hand, large dimensional errors associated with the locating holes of an incoming product may lead to locating tool failures such as a locating pin being broken during the part loading process, a part being stuck at pins, or a part being unable to be positioned correctly by the locators. In this paper, these kinds of locating tool failures are called locating tool failures induced by the incoming product quality. Therefore, the catastrophic failure rates of the locating tools are affected by the dimensional accuracy of the incoming product, which is in turn determined by the propagation of the dimensional product quality from the previous stations. Based on a previous study in Yang *et al.* (2000), the locating tool failure induced by the incoming product quality corresponds to about 44% of all locating tool catastrophic failures. In this paper, the reliability of a BIW assembly process with 3-2-1 fixtures and rigid parts is studied. The rigid part assembly covers 68% of the total parts in a typical autobody (Shiu *et al.*, 1997).

The product variation is propagated in a multi-station assembly process (Jin and Shi, 1999; Ding *et al.*, 2000). The final product quality is affected by the accumulation

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or stack up of all variations generated at previous stations. The variation propagation in product quality will lead to the propagation of the interaction between the locating tool reliability and the product quality, which is called the Quality and Relability (QR) chain effect. This paper will study the QR chain effect in a multi-station BIW assembly process.

A general QR chain modeling framework has been recently proposed by Chen *et al.* (2001) based on a *process model*, which is in the form of a linear regression model describing the relationship between process variables and product quality in manufacturing processes. The *process model* plays a critical role in analyzing the QR chain effect of a multi-station manufacturing process, which is assumed available in Chen *et al.* (2001). However, it is sometimes not easy to obtain a *process model* when design of experiments is not applicable, especially for a complex multistage manufacturing process. This paper will propose a procedure to build a *process model* for a multi-station BIW assembly process based on the engineering first principles. Once a *process model* has been obtained using process knowledge, the QR chain model of Chen *et al.* (2001) can be successfully applied to the BIW assembly process. All parameters used in system reliability analysis based on the QR chain model will have a corresponding physical meaning. In addition, an application of the QR chain model for the optimal assignment of locator wear rate is studied in this paper to improve the design of BIW assembly processes.

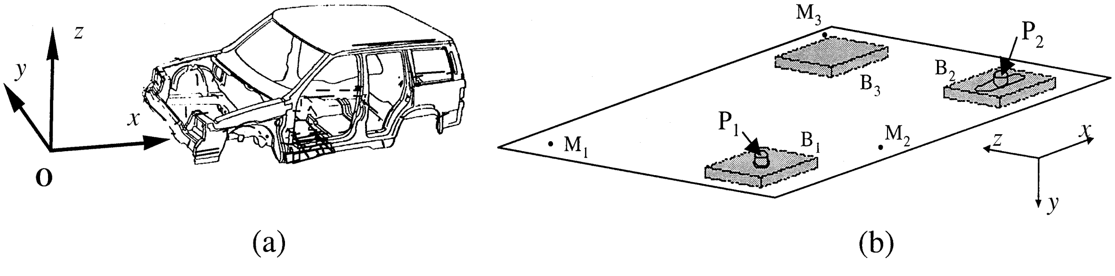
The *process model* of an assembly process should be obtained from product and process design information and a physical model for specific processes that considers variation propagation. In recent years, fixture systems and variation propagation in assembly processes have been studied and significant results have been achieved. The statistical description of variation patterns and the diagnostic issues of a fixture system have been addressed (Hu and Wu, 1992; Ceglarek *et al.*, 1994; Ceglarek and Shi, 1996; Apley and Shi, 1998). For multiple station assembly processes, Jin and Shi (1999) developed a state space model to describe the product variation propagation across different stations. The use of this state space model has enabled progress to be made in many areas including fault diagnosis (Ding *et al.*, 2000; Ding *et al.*, 2002; Zhou *et al.*, 2003), optimal sensor distribution (Ding *et al.*, 2003a), and optimal process tolerance design (Ding *et al.*, 2003b). However, there are few reports in the literature that concern assembly system reliability or the interactions between the product quality and locating tool reliability. In Jin and Chen (2001), the interaction between product quality and locating tool reliability were studied for a single assembly station. The propagation of the product quality and the QR interaction in a multi-station assembly process were not considered by Jin and Chen (2001). In general, the QR interaction in a multi-station assembly process is essentially much more complex than that in a single station process. With the aid of the state space model developed in Jin and Shi (1999) to describe the variation propagation, this paper will develop the corresponding *process model* for a multi-station BIW assembly process and then apply the QR chain model.

The paper is organized as follows. The process variables and product quality characteristics in the context of BIW assembly processes are specified in Section 2. In Section 3, the process model and other elements of the QR chain model are provided and the physical meanings of the model parameters are discussed in the context of a BIW assembly process. The system reliability is evaluated in Section 4 based on the QR chain model for a BIW assembly process. The application of the QR chain model is discussed in Section 5 to optimize the pin wear rate assignment. A case study is conducted in Section 6 to demonstrate the proposed model and methodology. The paper is concluded in Section 7.

2. Review of BIW assembly processes

2.1. *Fixture layout and locating principle in BIW assembly processes*

The body coordinate system shown in Fig. 1 is used in this paper. The origin of the body coordinate system is defined at the front center of a vehicle and below its underbody. The x, y and z axes are shown in the figure. This definition



(a) (b)

Fig. 1. Automotive body and its assembly fixture: (a) body coordinate system; and (b) the 3-2-1 fixturing principle for a rigid part.

of the body coordinate system has been widely used in the automotive industry for product and process design.

An assembly station typically consists of two or more assembly fixtures. Each fixture holds a single part to be assembled with other parts. Locating pins and blocks are locating tools that are widely used as fixtures to determine the part location and orientation. In this paper, a 3-2-1 fixture for rigid parts is assumed for each station. As shown in Fig. 1(b), a typical 3-2-1 fixture contains several key components: (i) a four-way pin/hole (P_1) to precisely locate a part in the x and z directions; (ii) a two-way pin/slot (P_2) to locate a part in the z-direction; these two pins constrain the part rotation and translation in the xz-plane together; and (iii) three shaded locating blocks (B_1, B_2, B_3) to locate a part in the y-direction. The combination of the locating tools (pins and blocks) constrains all six degrees-of-freedom of a rigid part. Since the degree of wear-out of locating blocks is very slight compared to that of locating pins, in this paper we only consider the locating pins in the BIW assembly processes.

2.2. *Major elements in the QR chain framework of a BIW assembly process*

2.2.1. *Process variables associated with locating pins*

A general modeling procedure focusing on the xy-plane is presented in this paper for rigid part assembly and the four-way and two-way locating pins are considered as being the system components. Suppose there are n_i locating pins at the ith station, $i = 1, 2, \ldots, L$, where L is the number of stations in a BIW assembly process. The total number of locating pins in all stations is $n = \sum_{i=1}^{L} n_i$. Any changes in the pin diameter will change the clearance between the locating pin on the fixture and the locating hole on the part, which will affect the product quality. Thus, the continuous decrease in the pin diameter caused by pin wear-out is considered to be a process variable. Let $P_{i,j}$ denote the jth locating pin at the ith station and $X_{i,j}(t)$ denote its accumulated diameter decrease at time t.

2.2.2. *Product quality characteristics in a BIW assembly process*

In assembly processes, product quality is generally defined by the dimensional accuracy of the Key Product Characteristic (KPC) points on the product. The KPC points on the outgoing product of each station include the locating holes used for part locating in the next station and the points whose dimensional accuracy is specified in the product design. The measurements of these KPC points are treated as the product characteristics in a BIW assembly process. Let $Y_{i,j}(t)$, $i = 1, \ldots, L$, $j = 1, \ldots, m_i$ denote the jth product characteristic on the outgoing product of station i at time t, where m_i is the number of KPC measurements on the outgoing product of station i.

3. QR chain model for BIW assembly processes

There are several key relationships in the QR chain model of a BIW assembly process: (i) the wear on the locating pin is the major cause of KPC deviations; (ii) the product quality of each station is defined based on the KPC deviations; and (iii) the outgoing product quality of a station impacts on the failure of the locating pin of the next station. The situation can be summerized as:

Pin degradation → KPC deviations → product quality → pin catastrophic failure. The following discussions will address these factors.

3.1. *Locating pin degradation model*

The mechanism of the wear on the locating pin is discussed in Jin and Chen (2001). They suggest that, the aggregate wear of the pin diameter increases with the number of operations. This can be described by a stochastic process model with independently lognormal distributed increments:

$$X_{i,j}(t) = X_{i,j}(t-1) + \Delta_{i,j}(t),$$

where $\Delta_{i,j}(t)$ is the increase in the random wear due to operation t, $X_{i,j}(0)$ is the initial clearance between locating pin $P_{i,j}$ and its corresponding locating hole, t is the operation index. It is assumed that $X_{i,j}(0) \sim \mathrm{N}(\mu_{i,j}(0), \sigma_{i,j}^2(0))$ and

$$\Delta_{i,j}(t) \sim \text{lognor}\,(\mu_{i,j}(\Delta), \sigma_{i,j}^2(\Delta)),$$

where $\mu_{i,j}(\Delta)$ and $\sigma_{i,j}(\Delta)$ are the mean and standard deviation of the lognormal random variable $\Delta_{i,j}(t)$. Let $\boldsymbol{\mu}_0 \equiv [\mu_{1,1}(0)\ \ \mu_{1,2}(0)\ \ \ldots\ \ \mu_{L,n_L}(0)]^T$ and

$$\boldsymbol{\Sigma}_0 \equiv \begin{bmatrix} \sigma_{1,1}^2(0) & & & \mathbf{0} \\ & \sigma_{1,2}^2(0) & & \\ & & \ddots & \\ \mathbf{0} & & & \sigma_{L,n_L}^2(0) \end{bmatrix}.$$

In this paper, we choose one production day as the time interval to discretize the time scale for a BIW assembly process. Let h denote the number of operations during each production day, and t_k denote the end time of the kth production day. Since the time is measured by the number of operations, t_k is the total number of operations until the end of production day k. So $t_k = kh$ and $t_0 = 0$. The BIW assembly operations are discrete in nature. The time for sliding wear when a part is positioned on the pin is much shorter than the cycle time of an operation (more time is spent on welding, clamping operations and part handling and moving). Also, the accumulated wear of a locating pin is much smaller than the pin diameter and has little impact on any future wear mechanism. Thus, it is reasonable to assume that the amount of wear during an operation is independent of that due to previous operations. In addition, a BIW assembly process can produce 500–1500 car bodies during each day of production. Therefore, the accumulated wear due to this large number of operations can reasonably be approximated as being normally distributed based on the central limit theorem. Thus, the following equation can be used to model the pin wear:

$$\mathbf{X}(t_k) = \mathbf{X}(t_{k-1}) + \varepsilon_k, k = 1, 2, \ldots, \tag{1}$$

where $\varepsilon_k \sim \mathrm{N}(\boldsymbol{\mu}_\varepsilon, \mathbf{Q})$, $\boldsymbol{\mu}_\varepsilon = h[\mu_{1,1}(\Delta) \ldots \mu_{1,n_1}(\Delta) \ldots \mu_{L,1}(\Delta) \ldots \mu_{L,n_L}(\Delta)]^T$, $\mathbf{Q} = h \times \mathrm{diag}\,(\sigma^2_{1,1}(\Delta) \ldots \sigma^2_{1,n_1}(\Delta) \ldots \sigma^2_{L,1}(\Delta) \ldots \sigma^2_{L,n_L}(\Delta))$, and $\mathbf{X}(t_0) \sim \mathrm{N}(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$. In this paper, increasing wear is assumed, that is, for any $1 \le j \le n$, $\Pr\{[\mathbf{X}(t_0)]_j < 0\}$ and $\Pr\{[\mathbf{X}(t_{k+1}) - \mathbf{X}(t_k)]_j < 0\}$ can be ignored.

3.2. *Relationship between process variables and KPC deviations*

The following process model is used in Chen *et al.* (2001) to describe the relationship between the process variables and the product quality characteristics:

$$Y_{i,j}(t) = \eta_{i,j} + \boldsymbol{\alpha}_{i,j}^T \mathbf{X}(t) + \boldsymbol{\beta}_{i,j}^T \mathbf{z}_t + \mathbf{X}(t)^T \boldsymbol{\Gamma}_{i,j} \mathbf{z}_t, \quad i = 1, 2, \ldots, L, j = 1, 2, \ldots, m_i, \tag{2}$$

where $\mathbf{X}(t) \equiv [X_{1,1}(t) X_{1,2}(t) \ldots X_{L,n_L}(t)]^T \in R^n$ is the vector of process variables in the system, $\mathbf{z}_t \equiv [z_{1t}, z_{2t}, \ldots, z_{lt}]^T \in R^l$ is the random vector of noise variables, with mean $E(\mathbf{z}_t)$ and covariance matrix $\mathrm{cov}(\mathbf{z}_t)$ independent of the time index t, l is the total number of noise variables in the BIW assembly process, $\eta_{i,j}$, $i = 1, 2, \ldots, L, j = 1, 2, \ldots, m_i$ are constants, $\boldsymbol{\alpha}_{i,j}$ and $\boldsymbol{\beta}_{i,j}$ are vectors characterizing the effects of $\mathbf{X}(t)$ and $\mathbf{z}_t$, and $\boldsymbol{\Gamma}_{i,j}$ is a matrix characterizing the effects of the interaction between $\mathbf{X}(t)$ and $\mathbf{z}_t$. For BIW assembly processes, $\mathbf{z}_t$ describes the random pin-hole contact orientations.

The physical model of a BIW assembly process needs to be studied to obtain the coefficients $\boldsymbol{\alpha}_{i,j}$, $\boldsymbol{\beta}_{i,j}$, $\boldsymbol{\Gamma}_{i,j}$ and $\eta_{i,j}$, $i = 1, 2, \ldots, L, j = 1, 2, \ldots, m_i$ in Equation (2). The relationship between process variables and product quality characteristics for a single fixture station is given in Jin and Chen (2001) by studying the relationship among locating pin diameters, part locating errors, and KPC deviations. An overall multi-station process model addressing the variation propagations across different stations is required in the development of the process model for a multi-station BIW assembly process. The state space model developed in Jin and Shi (1999) for multi-station sheet metal assembly processes will be used in this paper to obtain the coefficients of the process model. In the following sections, the use of a state space model and physical knowledge of the assembly process to obtain the coefficients of the process model for a BIW assembly process will be reviewed.

3.2.1. *State space model for part locating errors and KPC deviations*

From Jin and Shi (1999), the state equation at station i can be expressed by:

$$\mathbf{V}_i(t) = \mathbf{H}_{i-1}\mathbf{V}_{i-1}(t) + \mathbf{B}_i\mathbf{F}_i(t) \tag{3a}$$

$$\mathbf{Y}_i(t) = \mathbf{C}_i\mathbf{V}_i(t) \qquad i = 1, 2, \ldots, L, \tag{3b}$$

where $\mathbf{V}_i(t)$, which is equivalent to the state vector in the state space model, is the part error vector defined in Jin and Shi (1999) and characterizes the dimensional errors of all outgoing parts of station i, $\mathbf{V}_0(t)$ is the dimensional errors of the raw parts coming from the stamping processes, system matrices $\mathbf{H}$, $\mathbf{B}$ and $\mathbf{C}$ encode the process configurations such as the layout of locating tools and KPC points, $\mathbf{F}_i(t)$ is the vector of part locating errors which is the dimensional error of the part at the position of the locating pins of station i, and $\mathbf{Y}_i(t) \equiv [Y_{i,1}(t) \ldots Y_{i,m_i}(t)]^T$. Equation (3a) of the state space model uses a recursive relationship to characterize the propagation of product quality. Based on this relationship, it can be seen that the part dimensional errors at the current station are an accumulation of the part locating errors and raw part dimensional errors at the previous stations.

3.2.2. *Relationship between pin wear and part locating errors*

The wear of the locating pins is reflected in a reduction in the pin diameters, which causes an increasing clearance between a locating pin and the corresponding locating hole. This clearance results in the part locating error. The following notations are used in the description of the relationship between the pin wear and the part locating error:

(i) $\Delta x_{P_{i,j}}$ and $\Delta z_{P_{i,j}}$ denote the part locating errors of pin $\mathrm{P}_{i,j}$ in the x and z directions;
(ii) $\mathbf{F}_i(t) \equiv [\Delta x_{P_{i,1}} \ \Delta z_{P_{i,1}} \ \ldots \ \Delta x_{P_{i,n_i}} \ \Delta z_{P_{i,n_i}}]^T$ represents the vector of part locating errors at station i; and
(iii) $\theta_{i,j}$ represents the orientation of the contacting point between the pin $\mathrm{P}_{i,j}$ and the locating hole.

The relationship between the part locating errors $(\Delta x_{P_{i,j}}, \Delta z_{P_{i,j}})$ and the pin diameter reduction of a four-way locating pin can be obtained as:

$$\Delta x_{P_{i,j}} = 0.5 X_{i,j} \cos\theta_{i,j}, \quad \Delta z_{P_{i,j}} = 0.5 X_{i,j} \sin\theta_{i,j}, \tag{4}$$

where $X_{i,j}$ is the accumulated decrease in the pin diameter, which is considered to be a process variable that corresponds to the locating pin $\mathrm{P}_{i,j}$. The relationship between the part locating error and the wear of the two-way locating pin can be obtained as:

$$\Delta z_{P_{i,j}} = 0.5 X_{i,j} \sin\theta_{i,j}. \tag{5}$$

A more detailed illustration of Equations (4) and (5) and more discussions on the distribution of $\theta_{i,j}$ are given in Appendix 1.

3.2.3. *Process model for the BIW assembly process*

By recursively substituting $\mathbf{V}_i(t), \mathbf{V}_{i-1}(t), \ldots, \mathbf{V}_1(t)$ in Equation (3), the KPC deviation on the outgoing parts of station i can be calculated based on both the part locating errors at station i and those of previous stations as:

$$\mathbf{Y}_i(t) = \mathbf{G}^{(i)}\mathbf{F}^{(i)}(t) + (\mathbf{C}_i\mathbf{H}_{i-1}\mathbf{H}_{i-2}\cdots\mathbf{H}_0)\mathbf{V}_0(t), \tag{6}$$

where matrix $\mathbf{G}^{(i)} \equiv \mathbf{C}_i[(\mathbf{H}_{i-1}\mathbf{H}_{i-2}\cdots\mathbf{H}_1)\mathbf{B}_1\ (\mathbf{H}_{i-1}\mathbf{H}_{i-2}\cdots\mathbf{H}_2)\mathbf{B}_2\ \ldots\mathbf{H}_{i-1}\mathbf{B}_{i-1}\ \mathbf{B}_i]$ and vector $\mathbf{F}^{(i)}(t) \equiv [\mathbf{F}_1^T(t)\ \mathbf{F}_2^T(t)\ \ldots\ \mathbf{F}_i^T(t)]^T$.

Let $\mathbf{z}'_{t,i} \equiv [\cos\theta_{i,1}\ \sin\theta_{i,1}\ \ldots\ \cos\theta_{i,n_i}\ \sin\theta_{i,n_i}]^T$, $\mathbf{z}'_t \equiv [(\mathbf{z}'_{t,1})^T\ \ldots\ (\mathbf{z}'_{t,L})^T]^T$ and

$$\mathbf{z}_t \equiv \left[\mathbf{V}_0^T(t)\ (\mathbf{z}'_t)^T\right]^T. \tag{7}$$

That is, the raw part error and the random orientation of the contacting point between the pin and the locating hole are considered as noise variables in the process model. From Equations (6), (4) and (5), with some basic algebraic manipulations, it can be seen that:

$$\begin{aligned} Y_{i,j}(t) &= [\boldsymbol{\beta}_{\mathbf{i,j}}^{(1)}\ \boldsymbol{\beta}_{\mathbf{i,j}}^{(2)}]\mathbf{z}_t + \mathbf{X}(t)^T[\boldsymbol{\Gamma}_{i,j}^{(1)}\ \boldsymbol{\Gamma}_{i,j}^{(2)}]\mathbf{z}_t, \\ i &= 1, 2, \ldots, L, \quad j = 1, 2, \ldots, m_i, \end{aligned} \tag{8}$$

where $\boldsymbol{\beta}_{\mathbf{i,j}}^{(1)} = [\mathbf{C}_i\mathbf{H}_{i-1}\mathbf{H}_{i-2}\cdots\mathbf{H}_0]_{(j,:)}$, $\boldsymbol{\beta}_{\mathbf{i,j}}^{(2)}$ is a $1 \times 2n$ vector whose elements are all zero, $\boldsymbol{\Gamma}_{i,j}^{(1)}$ is a zero matrix of appropriate dimension; and

$$\boldsymbol{\Gamma}_{i,j}^{(2)} = \frac{1}{2}\begin{bmatrix} [\mathbf{G}^{(i)}]_{(j,1)} & [\mathbf{G}^{(i)}]_{(j,2)} & 0 & 0 & \cdots & 0 & 0 & \mathbf{0}_{(1\times 2(n-p_i))} \\ 0 & 0 & [\mathbf{G}^{(i)}]_{(j,3)} & [\mathbf{G}^{(i)}]_{(j,4)} & \cdots & 0 & 0 & \mathbf{0}_{(1\times 2(n-p_i))} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & [\mathbf{G}^{(i)}]_{(j,2p_i-1)} & [\mathbf{G}^{(i)}]_{(j,2p_i)} & \mathbf{0}_{(1\times 2(n-p_i))} \\ \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{((n-p_i)\times 1)} & \cdots & \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{((n-p_i)\times 1)} & \mathbf{0}_{(r\times 2(n-p_i))} \end{bmatrix}_{(n\times 2n)},$$

where $p_i \equiv \sum_{k=1}^{i} n_k$. Thus, the coefficients of the process model of Equation (2) for a BIW assembly process are $\eta_{i,j} = 0$, $\boldsymbol{\alpha}_{i,j} = \mathbf{0}$, $\boldsymbol{\beta}_{\mathbf{i,j}} = [\boldsymbol{\beta}_{\mathbf{i,j}}^{(1)}\ \boldsymbol{\beta}_{\mathbf{i,j}}^{(2)}]^T$ and $\boldsymbol{\Gamma}_{i,j} = [\boldsymbol{\Gamma}_{i,j}^{(1)}\ \boldsymbol{\Gamma}_{i,j}^{(2)}]$.

3.3. *Product quality assessment*

The product quality can be assessed by the mean-squared deviations of the quality characteristics: the KPCs. Therefore, the jth quality index of the ith station is:

$$\begin{aligned} q_{i,j}(t \mid \mathbf{X}(t_k)) &= E((Y_{i,j}(t) - \gamma_{i,j})^2 \mid \mathbf{X}(t_k)) \\ &= \mathrm{Var}(Y_{i,j}(t) \mid \mathbf{X}(t_k)) + (E(Y_{i,j}(t) \mid \mathbf{X}(t_k)) - \gamma_{i,j})^2, \\ &\quad t_k \le t < t_{k+1}, \end{aligned}$$

where $\gamma_{i,j}$ is the target value for $Y_{i,j}(t)$.

From the discussion in Section 3.2 and Appendix 1, it can be shown that $E(\mathbf{z}'_t) = \mathbf{0}$, thus $\boldsymbol{\Gamma}_{i,j}E(\mathbf{z}_t) = \mathbf{0}$ and $\boldsymbol{\alpha}_{i,j} + \boldsymbol{\Gamma}_{i,j}E(\mathbf{z}_t) = \mathbf{0}$. Then from Equation (2):

$$\begin{aligned} &(E(Y_{i,j}(t) \mid \mathbf{X}(t_k)) - \gamma_{i,j})^2 \\ &\quad = \mathbf{X}(t_k)^T(\boldsymbol{\alpha}_{i,j} + \boldsymbol{\Gamma}_{i,j}E(\mathbf{z}_t))(\boldsymbol{\alpha}_{i,j} + \boldsymbol{\Gamma}_{i,j}E(\mathbf{z}_t))^T\mathbf{X}(t_k) \\ &\quad = 0, \forall i, j, \text{and } k. \end{aligned}$$

Thus, the locating pin degradation of a BIW assembly process will not cause mean shifts of the KPCs. The quality index can be written as a quadratic function of $\mathbf{X}(k)$ as follows:

$$\begin{aligned} &q_{i,j}(t \mid \mathbf{X}(t_k)) \\ &\quad = \mathrm{var}(Y_{i,j}(t) \mid \mathbf{X}(t_k)) \\ &\quad = \left(\mathbf{X}(t_k)^T\boldsymbol{\Gamma}_{i,j}\right)\mathrm{cov}(\mathbf{z}_t)\left(\boldsymbol{\Gamma}_{i,j}^T\mathbf{X}(t_k)\right) + \boldsymbol{\beta}_{\mathbf{i,j}}^T\mathrm{cov}(\mathbf{z}_t)\boldsymbol{\beta}_{\mathbf{i,j}}, \\ &\quad = \mathbf{X}(t_k)^T\mathbf{B}_{i,j}\mathbf{X}(t_k) + d_{i,j}, \end{aligned} \tag{9}$$

where $\mathbf{B}_{i,j} = \boldsymbol{\Gamma}_{i,j}\mathrm{cov}(\mathbf{z}_t)\boldsymbol{\Gamma}_{i,j}^T$ and $d_{i,j} = \boldsymbol{\beta}_{\mathbf{i,j}}^T\mathrm{cov}(\mathbf{z}_t)\boldsymbol{\beta}_{\mathbf{i,j}}$. Since $\boldsymbol{\beta}_{\mathbf{i,j}}$ is only related to the raw part errors, $d_{i,j}$ can be interpreted as the contribution of the raw part errors to the product quality at each station.

Let:

$$E_t^{\mathrm{q}} \equiv \bigcap_{i=1}^{L}\bigcap_{j=1}^{m_i}(q_{i,j}(\tau) \le a_{i,j}, \forall 0 \le \tau \le t),$$

where $a_{i,j}$ is the threshold of the specification for the jth KPC at the ith station. E_t^{q} represents the event that no quality index has exceeded its threshold by time t, i.e., no failure due to nonconforming products has occurred by time t.

3.4. *Relationship between the product quality and the locating pin catastrophic failure rate*

Any deviation in the center of the locating hole of an incoming part may accelerate the catastrophic failure of its corresponding locating pin. The larger the variation in the locating hole position, the higher is the chance that the pin will fail. The relationship between the catastrophic failure rate $\lambda_{i,j}(t)$ and the product quality index $\mathbf{q}(t)$ can be described as:

$$\lambda_{i,j}(t) = \lambda_{i,j}(0) + \mathbf{s}_{i,j}^T\mathbf{q}(t \mid \mathbf{X}(t_k)),$$

where $\mathbf{q}(t \mid \mathbf{X}(t_k)) = [q_{1,1}(t \mid \mathbf{X}(t_k))\ q_{1,2}(t \mid \mathbf{X}(t_k))\ \ldots\ q_{L,m_L}(t \mid \mathbf{X}(t_k))]^T$ and $\mathbf{s}_{i,j}$ is a vector of appropriate dimension with non-negative elements and is called the QR coefficient. The QR coefficient can be calibrated by collecting catastrophic failure data and product dimensional measurements.

The catastrophic failure rate of a four-way locating pin can be affected by the locating hole variation in both the

x-direction and the z-direction. The catastrophic failure rate of a two-way locating pin is affected only by the locating hole variation in the z-direction since there is normally little contact in the x-direction between a locating hole and a two-way locating pin. Based on the relationship between the locating pin catastrophic failure rate and its corresponding locating hole variations discussed above, if $\mathrm{P}_{i,j}$ is a four-way pin, then:

$$[\mathbf{s}_{i,j}]_r = \begin{cases} \geq 0, & \text{if } [\mathbf{q}(t)]_r \text{ corresponds to the } x\text{- or} \\ & z\text{-direction variation of the locating hole} \\ & \text{located by the } j\text{th pin at the } i\text{th station,} \\ = 0, & \text{otherwise.} \end{cases}$$

If $\mathrm{P}_{i,j}$ is a two-way pin, then:

$$[\mathbf{s}_{i,j}]_r = \begin{cases} \geq 0, & \text{if } [\mathbf{q}(t)]_r \text{ corresponds to the } z\text{-direction} \\ & \text{variation of the locating hole located by} \\ & \text{the } j\text{th pin at the } i\text{th station,} \\ = 0, & \text{otherwise.} \end{cases}$$

Here $[\cdot]_r$ denotes the rth element of a vector.

4. System reliability evaluation

From the definition of $\mathbf{\Gamma}_{i,j}$ and $\mathbf{z}_t$ in Section 3.2.3 and the distribution of $\theta_{i,j}$ discussed in Appendix 1, it can be shown that $\mathbf{B}_{i,j} = \mathbf{\Gamma}_{i,j}\mathrm{cov}(\mathbf{z}_t)\mathbf{\Gamma}_{i,j}^T$ is diagonal. Based on Chen *et al.* (2001), the product quality of such a process does not have self-improvement. Therefore, following the same procedures in Chen *et al.* (2001), the system reliability of a BIW assembly process can be finally obtained as:

$$\begin{aligned} R(t_K) &\equiv \mathrm{Pr}(\text{no pin catastrophic failure AND each quality} \\ &\qquad \text{index is within specification by } t_K), \\ &= \mathrm{Pr}(E_{t_K}^{\mathrm{c}} \text{ AND } E_{t_K}^{\mathrm{q}}) = \mathrm{Pr}(E_{t_K}^{\mathrm{c}})\,\mathrm{Pr}(E_{t_K}^{\mathrm{q}} \mid E_{t_K}^{\mathrm{c}}), \\ &= R_{\mathrm{I}}(t_K) \times R_{\mathrm{II}}(t_K), \end{aligned} \tag{10}$$

where $t_K = Kh$ is the production mission time (the reliability evaluation lifetime),

$$R_{\mathrm{I}}(t_K) \equiv \exp(-ct_K)\frac{\exp(-\rho_K)}{|\mathbf{\Sigma}_K|^{\frac{1}{2}}}|\tilde{\mathbf{\Sigma}}_K|^{\frac{1}{2}},$$

$$R_{\mathrm{II}}(t_K) \equiv \int_{\mathbf{x}(t_K)\in\mathbf{\Omega}_K} \mathrm{d}F_{\tilde{\mathbf{X}}(t_K)}(\mathbf{x}(t_K)),$$

and E_t^{c} is defined as the event that system catastrophic failures have not occurred by time t. Because $R_{\mathrm{II}}(t_K)$ is affected by the quality constraint $\mathbf{\Omega}_K$, while $R_{\mathrm{I}}(t_K)$ is not, it can be seen that $R_{\mathrm{I}}(t_K) = \mathrm{Pr}(E_{t_K}^{\mathrm{c}})$, and $R_{\mathrm{II}}(t_K) = \mathrm{Pr}(E_{t_K}^{\mathrm{q}} \mid E_{t_K}^{\mathrm{c}})$. The probabilities associated with $E_{t_K}^{\mathrm{c}}$ and $E_{t_K}^{\mathrm{q}}$ are determined by the pin catastrophic failure rate derived in Section 3.4 and the quality index derived in Section 3.3. A detailed discussion on how to calculate $\mathrm{Pr}(E_{t_K}^{\mathrm{c}} \text{ AND } E_{t_K}^{\mathrm{q}})$ is presented in Chen *et al.* (2001). The parameters c, $\mathbf{\Sigma}_K$, $\mathbf{\Omega}_K$, ρ_K and the distribution of $\tilde{\boldsymbol{X}}(t_K)$ in Equation (10) are given in Appendix 2.

The following design parameters are used as the input information to calculate system reliability for general BIW assembly processes:

(a) Layout of the locating pins (used to get $\mathbf{H}_i$, $\mathbf{B}_i$, and $\mathbf{C}_i$ in the state space model).
(b) Layout of KPCs (used to get $\mathbf{C}_i$ in the state space model).
(c) Raw-part error ($\mathrm{cov}(\mathbf{V}_0(\mathrm{t}))$).
(d) Pin degradation rate and standard deviation ($\mu_{i,j}(\Delta)$, $\sigma_{i,j}(\Delta)$).
(e) Initial pin-hole clearance ($\mu_{i,j}(0)$, $\sigma_{i,j}(0)$).
(f) Initial catastrophic failure rate ($\lambda_{i,j}(0)$).
(g) QR coefficient $\mathbf{s}_{i,j}$.
(h) Threshold for quality index $a_{i,j}$.
(i) Length of time interval (h).

In Appendix 2, the procedure to calculate system reliability based on the input information is summarized in Table A1.

5. Optimal locator wear rate assignment

The analytical solution of Equation (10) has many potential applications in the design of a multi-station BIW assembly process. For example, it can be used to determine the optimal setting of the wear rates of the locating pins. Suppose that the wear rate range for each locating pin is defined as:

$$0 \leq \mu_{\min} \leq \mu_{i,j}(\Delta) \leq \mu_{\max} < \infty,$$

where $\mu_{\min}$ and $\mu_{\max}$ are the lowest and highest allowable wear rates for each locating pin. It should be noted that the selection of the pin wear rate directly affects its fabrication cost. Theoretically, zero is the lower bound for the wear rate. The $\mu_{\min}$ in our formulation is a practical lower bound. When $\mu_{\min} = 0$, it is equivalent to the problem without a specific practical lower bound. Therefore, the optimization without a lower bound (or lower bound is zero) is a special case of our model. We use $\mu_{\min}$ because in many situations if the wear rate is extremely small, it may not be achievable based on currently available tool making techniques. For these situations, a positive minimum value of $\mu_{\min}$ will be more reasonable in the optimization problem formulation.

5.1. *Fabrication and coating costs of locating pins*

Typically a lower wear rate of a locating pin comes at the cost of high fabrication and coating costs. Thus, it is reasonable to assume that the pin wear rate is inversely proportional to the fabrication and coating costs. In this paper, the costs associated with all locating pins are defined by a reciprocal function of the pin wear rates:

$$C_{\mathrm{p}} = \sum_{i=1}^{L}\sum_{j=1}^{n_i}\frac{w_{i,j}}{\mu_{i,j}(\Delta)}, \tag{11}$$

where $\mu_{i,j}(\Delta)$ is the wear rate of pin $\mathrm{P}_{i,j}$ and $w_{i,j} \geq 0$ is the weighting coefficient associated with $\mu_{i,j}(\Delta)$.

5.2. *Optimization formulation and optimality*

The objective of the optimal assignment of the locating pin wear rates is to maximize the system reliability at t_K. The optimization problem can be formulated mathematically as:

$$\boldsymbol{\mu}^* = \arg\max_{\boldsymbol{\mu}}\{R(t_K)\},$$

subject to

$$C_\mathrm{p} \le C_{\max}, \mu_{\min} \le \mu_{i,j}(\Delta) \le \mu_{\max}, \forall i, j, \tag{12}$$

where $\boldsymbol{\mu} = [\mu_{1,1}(\Delta)\ \mu_{1,2}(\Delta)\ \ldots\ \mu_{L,n_L}(\Delta)]^T$ and $C_{\max}$ is the budget for the total tooling fabrication cost.

Let $\bar{E}^\mathrm{q}_{t_K}$ denote the complement of $E^\mathrm{q}_{t_K}$. $\bar{E}^\mathrm{q}_{t_K}$ is the event that the failure due to nonconforming products occurs by time t_K. If we first constrain the maximum wear rate $\mu_{\max}$ such that $\Pr(\bar{E}^\mathrm{q}_{t_K}) < \alpha$, then the failure due to nonconforming products can be ignored. The selection of α depends on specific manufacturing processes. The BIW assembly process is not a highly reliable process due to its complexity. Normally the system reliability of interest for a BIW assembly process cannot be very high and a failure probability of less than 0.01 due to nonconforming products is acceptable. Therefore, for BIW assembly processes, we choose $\alpha = 0.01$. It can be seen that:

$$\begin{aligned} R(t_K) &= 1 - \Pr\left(\bar{E}^\mathrm{c}_{t_K} \cup \bar{E}^\mathrm{q}_{t_K}\right) \\ &= 1 - \Pr\left(\bar{E}^\mathrm{c}_{t_K}\right) - \Pr\left(\bar{E}^\mathrm{q}_{t_K}\right) + \Pr\left(\bar{E}^\mathrm{c}_{t_K} \cap \bar{E}^\mathrm{q}_{t_K}\right) \end{aligned}$$

Since $0 \le \Pr(\bar{E}^\mathrm{c}_{t_K} \cap \bar{E}^\mathrm{q}_{t_K}) \le \Pr(\bar{E}^\mathrm{q}_{t_K}) < \alpha$:

$$\left|R(t_K) - \left(1 - \Pr\left(\bar{E}^\mathrm{c}_{t_K}\right)\right)\right| < \alpha.$$

Therefore, if $\Pr(\bar{E}^\mathrm{q}_{t_K})$ can be ignored, the objective of maximizing $R(t_K)$ becomes maximizing:

$$1 - \Pr\left(\bar{E}^\mathrm{c}_{t_K}\right) = \Pr\left(E^\mathrm{c}_{t_K}\right) = R_\mathrm{I}(t_K).$$

Now suppose that the constrained maximum wear rate $\mu_{\max}$ is selected such that:

$$\Pr\left(\bar{E}^\mathrm{q}_{t_K} \mid \mu_{i,j}(\Delta) = \mu_{\max}, \forall i, j\right) < \alpha. \tag{13}$$

The following result ensures that under this condition, the failure due to nonconforming products can be ignored for all settings of the locating pin wear rates within the allowable range.

Result 1. *If* $\mu'_{i,j}(\Delta) \le \mu''_{i,j}(\Delta), \forall i, j,$ *then* $\Pr(\mathrm{E}_{\mathrm{t_K}}{}^{\mathrm{q}} \mid \mu_{i,j}(\Delta) = \mu'_{i,j}(\Delta)) \ge \Pr(\mathrm{E}_{\mathrm{t_K}}{}^{\mathrm{q}} \mid \mu_{i,j}(\Delta) = \mu''_{i,j}(\Delta))$.

Proof. This result can be easily seen based on the fact that the product quality of the BIW assembly process does not have self-improvement. The detailed proof is omitted in this paper. ■

From Equation (13) and Result 1, the failure due to nonconforming products can be ignored for any pin wear rate settings within the allowable ranges, and the optimization problem in Equation (12) is equivalent to the following optimization problem:

$$\boldsymbol{\mu}^* = \arg\max_{\boldsymbol{\mu}}\{R_\mathrm{I}(t_K)\},$$

subject to

$$C_\mathrm{p} \le C_{\max}, \mu_{\min} \le \mu_{i,j}(\Delta) \le \mu_{\max}, \forall i, j. \tag{14}$$

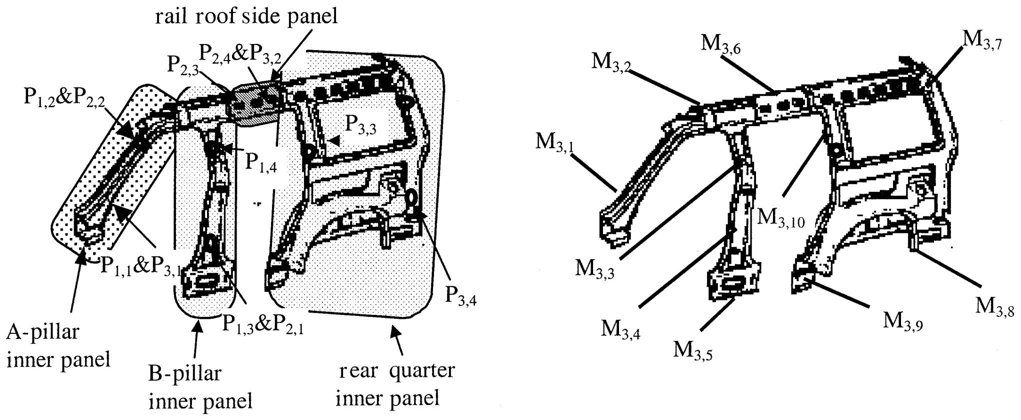
Since c, $\boldsymbol{\Sigma}_K$, and $\tilde{\boldsymbol{\Sigma}}_K$ are independent of the decision variable $\boldsymbol{\mu}$, Equation (14) can be further rewritten as:

$$\boldsymbol{\mu}^* = \arg\min_{\boldsymbol{\mu}}\{s_K\},$$

subject to

$$C_\mathrm{p} \le C_{\max}, \mu_{\min} \le \mu_{i,j}(\Delta) \le \mu_{\max}, \forall i, j. \tag{15}$$

Regarding the optimality of the above optimization formulation, the following results can be derived based on the form of the analytical solution.



(a) (b)

Fig. 2. Layout of: (a) the locating pin positions; and (b) the KPC positions.

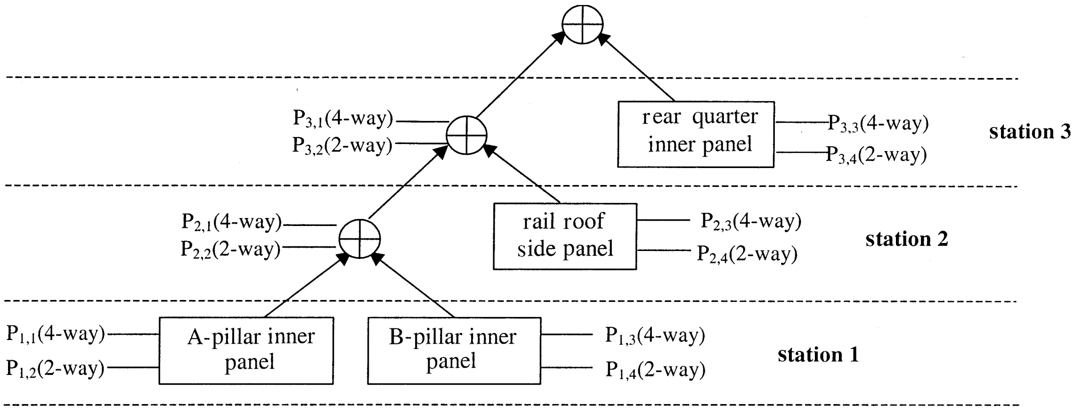


Fig. 3. Assembly sequences of the side aperture inner panel.

Lemma 1. *ρ_K is a convex function of $\boldsymbol{\mu}$.*

Proof. From Equation (A3), $\rho_K = \boldsymbol{\mu}_K^T[\mathbf{U}_K^T(\mathbf{U}_K + (\boldsymbol{\Sigma}_K^{-1}/2))^{-T}(\boldsymbol{\Sigma}_K^{-1}/2)]\boldsymbol{\mu}_K$. Both $\mathbf{U}_K$ and $\boldsymbol{\Sigma}_K$ are symmetric positive semidefinite matrices based on the discussion in Chen *et al.* (2001). Thus, the summation, product, and inverse of them are all positive semidefinite. Therefore, ρ_K is positive semidefinite on $\boldsymbol{\mu}_K$. From Equation (A3), obviously ρ_K is also positive semidefinite on $\boldsymbol{\mu}$. Hence, it is convex on $\boldsymbol{\mu}$. ■

Lemma 2. *The constraint in Equation* (15) *is a convex set.*

Proof. Obviously C_p is a convex function of $\mu_{i,j}(\Delta)$ for $\mu_{i,j}(\Delta) > 0$. So the level set $C_p \leq C_{\max}$ is convex and hence $\{\mu_{i,j}(\Delta) \mid C_p \leq C_{\max}, \mu_{\min} \leq \mu_{i,j}(\Delta) \leq \mu_{\max}\}$ is a convex set. ■

From Lemma 1 and Lemma 2, the result below about the optimality of Equation (15) follows (Avriel, 1976).

Result 2. *The nonlinear optimization problem stated in Equation* (15) *converges to a global minimum $\boldsymbol{\mu}^*$.*

6. Case study

A case study is conducted to illustrate the developed methodology. A side aperture inner panel assembly, as shown in Fig. 2 (a and b), is selected in the study.

In this example, four parts are assembled together by three stations (Fig. 3). A-pillar and B-pillar are assembled at station 1. The subassembly of A-pillar and B-pillar is then assembled with rail roof at station 2. At station 3, the subassembly of A-pillar, B-pillar, and rail roof are assembled with the rear quarter inner panel. The product quality is defined by 10 KPC points measured at station 3 (Fig. 2(b)). The assembly sequences of these four parts are illustrated in Fig. 3 and the layout of tooling positions is shown in Fig. 2(a). Table 1 and Table 2 give all the dimensions of the tooling positions and the KPC points. The design parameters used for this example are shown in Table 3. The raw part dimensional errors and variation of initial pin/hole clearance is very small and ignored in this case study.

In the study, two cases are investigated to demonstrate the concepts and potential applications of the proposed methodology.

Case 1. system reliability analysis with and without considering the QR chain effect: the system reliability analysis is conducted under three different definitions of system failures, which are:

(a) Only consider the probability of component catastrophic failures. That is, the pin degradation and the impact of the incoming product quality on the pin catastrophic failure are not considered in the model. It is equivalent to the case of setting the QR coefficient s and the pin degradation rate $\mu(\Delta)$ to zero in the QR chain model;
(b) Consider both the pin catastrophic failure and the product quality deterioration due to component wear-out, but without considering the impact of the incoming product quality on the catastrophic failure rate of the locating pins. It is equivalent to the case of setting the QR coefficient s in the QR chain model to zero;
(c) Consider the integrated QR chain model proposed in this paper.

Table 1. Nominal *x*-*z* coordinates for locating points

Locating points	$P_{1,1}$ & $P_{3,1}$	$P_{1,2}$ & $P_{2,2}$	$P_{1,3}$ & $P_{2,1}$	$P_{1,4}$	$P_{2,3}$	$P_{2,4}$ & $P_{3,2}$	$P_{3,3}$	$P_{3,4}$
Nominal coordinates (mm)								
x	367.7	667.5	1272.7	1301.0	1470.7	1770.5	2120.3	3026.3
z	906.1	1295.4	537.4	1368.9	1640.4	1702.6	1402.8	950.3

Table 2. Nominal *x*-*z* coordinates for KPC points

	KPC points									
	$M_{3,1}$	$M_{3,2}$	$M_{3,3}$	$M_{3,4}$	$M_{3,5}$	$M_{3,6}$	$M_{3,7}$	$M_{3,8}$	$M_{3,9}$	$M_{3,10}$
Nominal coordinates (mm)										
x	271.5	565	1289	1306	1244	1640	2884	2743	1838	1980
z	905.0	1634	1227	633	85	1781	1951	475	226	1459

The system reliability results obtained from the formulae in Section 4 and the calculation steps in Table A1 of Appendix 2 are shown in Fig. 4. The Matlab code for the numerical evaluation of the system reliability was run on an IBM PC Pentium III machine. On average, it takes about 10 seconds to evaluate the system reliability at a specific time based on the QR chain model. Therefore, the evaluation algorithm used in this paper is quite efficient and feasible. From the comparison study, it can be seen that the system reliabilities under definitions (a) and (b) are always higher than that under definition (c). The overestimation of definitions (a) and (b) is not surprising. Based on real production data, as discussed in the Introduction, about 44% of locating tool catastrophic failures in BIW assembly processes are induced by the incoming product quality. As a result, ignorance of the impact of the incoming product quality as in (a) and (b) may lead to significant overestimation of the overall system reliability. If a scheduled maintenance policy is planned based on the predicted system reliability using definition (a) or (b), many unexpected down times could be experienced due to neglecting the interdependency between the product quality and the reliability of locating pins.

Case 2. optimal setting of the wear rates of the locating pins: suppose a preferred preventive maintenance cycle is every 50 production days based on the production schedule. From the simulation result in Fig. 4, current system reliability at the 50th production day is only about 0.75. In order to reduce the unexpected system failure before the 50th production day, the designer needs to improve $R(t_{50})$. The overlap portion of the curves (a) and (b) in Fig. 4 shows that the system failure before the 50th production day is mainly caused by system catastrophic failure rather than failure due to nonconforming products. Thus, up to the 50th production day failure due to nonconforming products can be ignored (with a probability of less than 1%). Furthermore, the difference between curve (c) and curves (a) and (b) shows that by the 50th production day the incoming part quality has a significant impact on the catastrophic failure of locating pins. Therefore, the designer can improve $R(t_{50})$ by reducing the wear rates of the locating pins so that the incoming product quality of each station is improved and the QR interactions are reduced.

In case study 1, the wear rate of each locating pin is set to be 2×10^{-6} mm/operation. Assuming $w_{i,j} = w, \forall i,j$, the current pin fabricating cost can be calculated from Equation (11) as $C_0 = 12w/2 \times 10^{-6}$. Suppose that the designer wants to improve $R(t_{50})$ but that the available budget

Table 3. Summary of the parameters used in the study

Description of the parameter	*Value*
Initial failure rate $\lambda_{i,j}(0)$	$\lambda_{i,j}(0) = \lambda(0) = 4 \times 10^{-7}, \forall i,j$
QR coefficient $s_{i,j}$	$[s_{i,j}]_r = s = 0.001, \forall i,j,$ when the rth element corresponds to the quality characteristics of the locating hole located by the jth pin at the ith station
Initial pin/hole clearance $\mu_{i,j}(0)$	$\mu_{i,j}(0) = \mu_0 = 0.04$ mm, $\forall i,j$
Operations per time interval h	$h = 500$ operations
Unit degradation rate $\mu_{i,j}(\Delta)$	$\mu_{i,j}(\Delta) = \mu(\Delta) = 2 \times 10^{-6}$ mm/operation,$\forall i,j$
Threshold for quality index $a_{i,j}$	$6a_{i,j} = 6a = 0.8$ mm, $\forall i,j$
Unit degradation s.t.d. $\sigma_{i,j}(\Delta)$	$\sigma_{i,j}(\Delta) = \sigma(\Delta) = 5 \times 10^{-5}$ mm/operation,$\forall i,j$

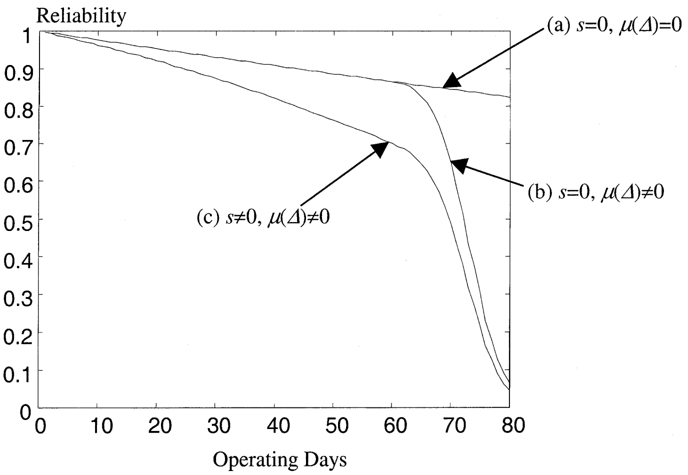


Fig. 4. System reliability with and without considering the QR chain effect.

Table 4. Optimal pin wear rates $\boldsymbol{\mu}^*$ based on the QR chain model (10^{-6} mm/operation)

$\mu_{1,1}(\Delta)$	$\mu_{1,2}(\Delta)$	$\mu_{1,3}(\Delta)$	$\mu_{1,4}(\Delta)$	$\mu_{2,1}(\Delta)$	$\mu_{2,2}(\Delta)$	$\mu_{2.3}(\Delta)$	$\mu_{2,4}(\Delta)$	$\mu_{3,1}(\Delta)$	$\mu_{3,2}(\Delta)$	$\mu_{3,3}(\Delta)$	$\mu_{3,4}(\Delta)$
0.71	0.80	0.51	2.0	0.78	0.63	1.54	0.73	2.0	2.0	2.0	2.0

only allows an increase in the pin fabrication cost by at most one-third. One simple way to do this is to reduce the wear rate of each locating pin from 2×10^{-6} to 1.5×10^{-6} mm/operation. However, the more efficient way is to spend more on reducing the wear rates of the critical locating pins determined by the QR chain model. For this purpose, an optimization problem can be formulated as:

$$\boldsymbol{\mu}^* = \arg \max_{\boldsymbol{\mu}} \{R(t_{50})\},$$

subject to

$$C_{\mathrm{p}} \leq \frac{4}{3} C_0, 0 \leq \mu_{i,j}(\Delta) \leq 2 \times 10^{-6}, \forall i, j. \tag{16}$$

This optimization problem is a special case of Equation (12) with $K = 50$, $C_{\max} = (4/3)C_0$, $\mu_{\min} = 0$ and $\mu_{\max} = 2 \times 10^{-6}$. Also, attention can be solely focused on system catastrophic failure since failure due to nonconforming products can be ignored up to the 50th production day.

The optimal pin wear rate assignment based on the discussion on the optimality of the optimization problem of Equation (12), is shown in Table 4. The optimization problem is solved by using the Matlab function *fmincon* that uses a sequential quadratic programming method (Anon, 1999). The algorithm converges within 5 seconds, which is pretty efficient. From Table 4, it can be seen that there is no need to improve pins $P_{1,4}$, $P_{3,1}$, $P_{3,2}$, $P_{3,3}$ and $P_{3,4}$ in the optimal solution. First, we have already calculated that the final product quality is satisfactory at t_{50} even if all the locating pin wear rates are not improved. Thus, leaving these five pins unimproved will not result in a poor final product quality. Secondly, in terms of reducing the pin catastrophic failures before t_{50} under the constraint that the final product quality is satisfactory, the obtained result is consistent with our physical understanding of the process design. From Figs. 2 and 3, pins $P_{3,1}$, $P_{3,2}$, $P_{3,3}$ and $P_{3,4}$ are used at station 3, which is considered to be the final station in this example and whose output is the final product rather than an incoming part for the next station. So the degradation of these four pins will not contribute to the pin catastrophic failures of later stations. Locating pin $P_{1,4}$ contributes to the rotation movement of the B-pillar around the four-way pin $P_{1,3}$. This movement does not affect the position of the locating hole for $P_{1,3}$, which is also used as the locating hole for $P_{2,1}$ at station 2. Thus, degradation of $P_{1,4}$ also does not contribute to pin catastrophic failure of later stations. Different degrees of improvements are performed for other locating pins based on their geometrical relationship and the part locating mechanism.

Table 5. Comparison of $1 - R(t_{50})$ for three different settings of pin wear rates

$\boldsymbol{\mu}$	*2×10^{-6} mm/ operation*	*1.5×10^{-6} mm/ operation*	$\boldsymbol{\mu}^*$
$F(t_{50}) = 1 - R(t_{50})$	0.237	0.214	0.182

Table 5 is used to compare the original system failure probability at t_{50} (which is $1 - R(t_{50})$), the improved failure probability based on uniform 1.5×10^{-6} mm/operation wear rates for each pins, and the improved failure probability based on the optimal solution $\boldsymbol{\mu}^*$. Setting the wear rates uniformly improves the system failure probability by 9.7%. By using the optimal wear rate setting based on the QR chain model, the system failure probability can be improved by 23.2%, with the pin fabrication cost remaining the same as that of the uniform wear rate setting. Therefore, with the aid of the QR chain model developed in this paper, an optimal design can be achieved to maximize the system reliability (or minimize the system failure probability) under the constraints of available budget on tooling fabrication costs.

7. Summary

Quality and reliability are two important factors in manufacturing system design. In BIW assembly processes, real production data have shown significant interactions between locating tool reliability and product quality. This paper developed a *process model* of a multi-station BIW assembly process based on the process knowledge. The QR chain model is applied based on the *process model* to capture the QR interaction and its propagation through multi-station BIW assembly processes. Process and product design information of assembly systems were integrated into the QR chain model and a state space model was employed to study the variation propagation through all assembly stations. An analytical solution for system reliability evaluation has been obtained based on the QR chain model.

A case study is conducted in this paper, which shows that the system reliability will be overestimated if the QR interaction is improperly ignored. An optimization problem, as a typical application of the QR chain model, is formulated to optimally assign pin wear rates with a constrained budget on pin fabrication costs. The optimality of the optimal solution is derived based on the analytical form of the system reliability. The optimal design obtained from the QR chain model significantly improves the system reliability compared with that obtained from current practice.

It should be noticed that future research is still needed on how to select and design techniques to achieve the assigned optimal wear rate for each locating pin. Another interesting future study following this paper is the integration of the maintenance decision-making and the wear rate design for locating pins, through which an overall optimal productivity of BIW assembly processes can be achieved.

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Appendices

Appendix 1. Details on relationship between the pin wear and part locating error

The relationship between the pin wear and the part locating error in a fixture station has been studied in Jin and Chen (2001). Based on both laboratory observations and data obtained in real autobody assembly plants, in most cases the locating pin touches the wall of the locating hole during the assembly operations. Thus, it is reasonable to assume that the locating hole contacts with the pin on one side when the part is positioned by a fixture. Due to the possibility that the locating pin does not touch the locating hole, this assumption may result in a slight overestimation of the product variation and lead to conservative predictions of the reliability.

From the assumption above, the contacting orientations between the locating pin and the locating hole for a four-way pin and a two-way pin are shown in Fig. A1. The part locating error in the xy-plane can be represented by the displacement of the locating hole center from the center of the pin as shown in Fig. A1. Based on Fig. A1, the relationship between the part locating errors $(\Delta x_{P_{i,j}}, \Delta z_{P_{i,j}})$ and the pin diameter reduction of a four-way pin can be obtained as

$$\Delta x_{P_{i,j}} = 0.5X_{i,j}\cos\theta_{i,j}; \quad \Delta z_{P_{i,j}} = 0.5X_{i,j}\sin\theta_{i,j} \tag{A1}$$

Here $\theta_{i,j}, \forall i, j$ are assumed to be independent random variables following a uniform distribution within $[0, 2\pi]$, which is denoted as $\theta_{i,j} \sim \mathrm{U}(0, 2\pi)$. It can also be shown that $\mathrm{var}(\sin\theta_{i,j}) = \mathrm{var}(\cos\theta_{i,j}) = 0.5$ and $\mathrm{cov}(\sin\theta_{i,j}, \cos\theta_{i,j}) = 0$ for a four-way locating pin. Similarly, ignoring the effect of the wear of a two-way pin in the x-direction, the relationship between the part locating error and the wear of the two-way pin can be obtained as:

$$\Delta z_{P_{i,j}} = 0.5X_{i,j}\sin\theta_{i,j}. \tag{A2}$$

Since the locating hole contacts with the two-way pin either on the upper or the lower side in the z-direction, $\theta_{i,j}$ is a random variable having two values of $-\pi/2$ and $\pi/2$ with the same probability of one-half. In this paper, this is denoted as $\theta_{i,j} \sim \mathrm{Unif}\,\{-\pi/2, \pi/2\}$ for two-way locating pins. It can also be shown that $\mathrm{var}(\sin\theta_{i,j}) = 1$ for a two-way locating pin.

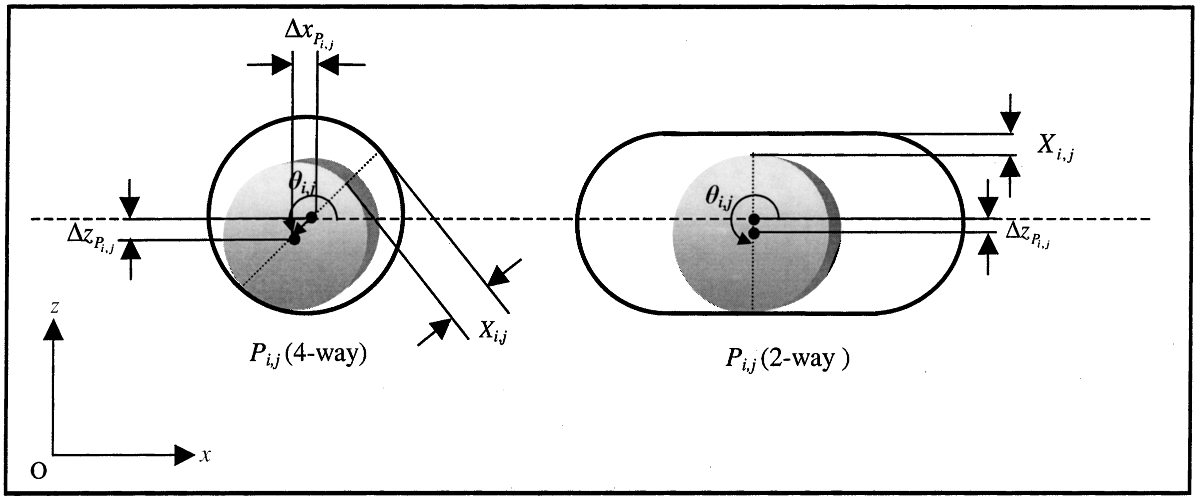

Fig. A1. Part locating error due to pin wear.

Appendix 2. Parameters in equation (10) and procedure to calculate system reliability

Based on the reliability results for a general manufacturing process with a QR chain given in Chen *et al.* (2001), the parameters in Equation (10) are:

(i) $c \equiv \sum_{i=1}^{L} \sum_{j=1}^{n_i} \lambda_{i,j}(0) + \sum_{i=1}^{L} \sum_{j=1}^{n_i} \mathbf{s}_{i,j}^{T}\mathbf{d}$, where $\mathbf{d} \equiv d_{1,1}\ d_{1,2}\ \ldots\ d_{L,m_L-1}\ d_{L,m_L}]^T$, $d_{i,j}$ is obtained in Equation (9).

(ii)

$$\Sigma_K \equiv \begin{bmatrix} \boldsymbol{\Sigma}(0) & \boldsymbol{\Sigma}(0,1) & \ldots & \boldsymbol{\Sigma}(0,K) \\ \boldsymbol{\Sigma}(1,0) & \boldsymbol{\Sigma}(1) & \ldots & \boldsymbol{\Sigma}(1,K) \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\Sigma}(K,0) & \boldsymbol{\Sigma}(K,1) & \ldots & \boldsymbol{\Sigma}(K) \end{bmatrix}_{(n(K+1)\times n(K+1))}.$$

From Equation (1), $\boldsymbol{\Sigma}(k+i,k) \equiv \mathrm{cov}(\mathbf{X}(t_{k+i}), \mathbf{X}(t_k)) = \boldsymbol{\Sigma}(k)$, $\boldsymbol{\Sigma}(k,k+i) = \boldsymbol{\Sigma}(k+i,k)^T = \boldsymbol{\Sigma}(k)$, and $\boldsymbol{\Sigma}(k) = \boldsymbol{\Sigma}(k-1) + \mathbf{Q}$, $k = 1, 2, \ldots, \boldsymbol{\Sigma}(0) = \boldsymbol{\Sigma}_0$.

Table A1. General procedures to calculate system reliability of BIW assembly processes

Step	*Outputs*	*Inputs*	*Related formula/results in the paper*
1	$\mathrm{cov}(\mathbf{z}_t)$	(c)*	Equation (7) and Appendix 1
2	$\boldsymbol{\Gamma}_{i,j}$	(a) and (b)	Section 3.2.3 and Equation (3)
3	$\mathbf{Q}$ and Σ_0	(d) and (e)	Equation (1)
4	$\boldsymbol{\mu}_K$	(d) and (e)	Equation (A3)
5	$\mathbf{d}$	(a), (b), and $\mathrm{cov}(\mathbf{z}_t)$ from step 1	Equations (8) and (9)
6	$\mathbf{B}_{i,j}$	$\boldsymbol{\Gamma}_{i,j}$ from step 2 and $\mathrm{cov}(\mathbf{z}_t)$ from step 1	Equation (9)
7	c	(f), (g), and $\mathbf{d}$ from step 5	Appendix 2(i)
8	Σ_K	$\mathbf{Q}$ and Σ_0 from step 3	Appendix 2(ii)
9	$\boldsymbol{\Omega}_K$	(h), $\mathbf{d}$ from step 5, and $\mathbf{B}_{i,j}$ from step 6	Appendix 2(iii)
10	$\mathbf{U}_K$	(g), (i), $\mathbf{B}_{i,j}$ from step 6	Appendix 2(iv)
11	ρ_K	$\boldsymbol{\mu}_K$ from step 4, Σ_K from step 8, and $\mathbf{U}_K$ from step 10	Appendix 2(v)
12	$\tilde{\boldsymbol{\mu}}_K$	$\boldsymbol{\mu}_K$ from step 4, $\mathbf{U}_K$ from step 10, and Σ_K from step 8	Appendix 2(iv)
13	$\tilde{\boldsymbol{\Sigma}}_K$	Σ_K from step 8 and $\mathbf{U}_K$ from step 10	Appendix 2(iv)
14	$F_{\tilde{\mathbf{X}}(t_K)}$	$\tilde{\boldsymbol{\Sigma}}_K$ from step 13 and $\tilde{\boldsymbol{\mu}}_K$ from step 12	Appendix 2(iv)
15	$R_{\mathrm{I}}(t_K)$	c from step 7, ρ_K from step 11, Σ_K from step 8, and $\tilde{\boldsymbol{\Sigma}}_K$ from step 13	Equation (10)
16	$R_{\mathrm{II}}(t_K)$	$\boldsymbol{\Omega}_K$ from step 9 and $F_{\tilde{\mathbf{X}}(t_K)}$ from step 14	Equation (10)
17	$R(t_K)$	$R_{\mathrm{I}}(t_K)$ from step 15 and $R_{\mathrm{II}}(t_K)$ from step 16	Equation (10)

*The input numbering in parenthesis follows that of the input list in Section 4.

(iii) $\boldsymbol{\Omega}_K$ is a domain in R^n subject to:

$$\mathbf{x}(t_K) \in \boldsymbol{\Omega}_K \Leftrightarrow \bigcap_{i=1}^{L} \bigcap_{j}^{m_i} \{\mathbf{x}^T(t_K)\mathbf{B}_{i,j}\mathbf{x}(t_K) \le a_{i,j} - d_{i,j}\}.$$

(iv) The distribution of $\tilde{\mathbf{X}}(t_K)$ is $\tilde{\mathbf{X}}(t_K) \sim \mathrm{N}(\tilde{\boldsymbol{\mu}}(K), \tilde{\boldsymbol{\Sigma}}(K))$, where $\tilde{\boldsymbol{\mu}}(K)$ and $\tilde{\boldsymbol{\Sigma}}(K)$ can be obtained by partitioning a matrix $\tilde{\boldsymbol{\Sigma}}_K$ and a vector $\tilde{\boldsymbol{\mu}}_K$ as:

$$\tilde{\boldsymbol{\Sigma}}_K = \left[\begin{array}{c|c} \underset{(nK\times nK)}{\boldsymbol{\Sigma}11} & \underset{(nK\times n)}{\boldsymbol{\Sigma}12} \\ \hline \underset{(n\times nK)}{\boldsymbol{\Sigma}21} & \underset{(n\times n)}{\boldsymbol{\Sigma}22} \end{array}\right], \tilde{\boldsymbol{\mu}}_K = \left[\begin{array}{c} \underset{(nK\times 1)}{\boldsymbol{\mu}1} \\ \hline \underset{(n\times 1)}{\boldsymbol{\mu}2} \end{array}\right],$$

and $\tilde{\boldsymbol{\mu}}(K) = \boldsymbol{\mu}2$ and $\tilde{\boldsymbol{\Sigma}}(K) = \boldsymbol{\Sigma}22$. From Chen *et al.* (2001), $\tilde{\boldsymbol{\mu}}_K$ and $\tilde{\boldsymbol{\Sigma}}_K$ can be calculated by:

$$\tilde{\boldsymbol{\mu}}_K \equiv \left(\mathbf{U}_K + \boldsymbol{\Sigma}_K^{-1}/2\right)^{-1}\left(\boldsymbol{\Sigma}_K^{-1}/2\right)\boldsymbol{\mu}_K, \tilde{\boldsymbol{\Sigma}}_K^{-1} \equiv 2\mathbf{U}_K + \boldsymbol{\Sigma}_K^{-1},$$

where

$$\mathbf{U}_K = \begin{bmatrix} \mathbf{I}_{(K\times K)} \otimes \mathbf{U}' & \mathbf{0}_{(nK\times n)} \\ \mathbf{0}_{(n\times nK)} & \mathbf{0}_{(n\times n)} \end{bmatrix},$$

$$\mathbf{U}' \equiv h \sum_{i=1}^{L} \sum_{j=1}^{n_i} \sum_{k=1}^{i-1} \sum_{l=1}^{m_k} [\boldsymbol{s}_{i,j}]_{q_{k-1}+l}\mathbf{B}_{k,l}$$

and $q_{k-1} \equiv \sum_{i=1}^{k-1} m_i$.

(v) ρ_K can be calculated as:

$$\rho_K = \boldsymbol{\mu}_K^T\left[\mathbf{U}_K^T\left(\mathbf{U}_K + \left(\boldsymbol{\Sigma}_K^{-1}/2\right)\right)^{-T}\left(\boldsymbol{\Sigma}_K^{-1}/2\right)\right]\boldsymbol{\mu}_K > 0, \tag{A3}$$

where $\boldsymbol{\mu}_K \equiv [\boldsymbol{\mu}^T(0)\ \ \boldsymbol{\mu}^T(1)\ \ \ldots\ \ \boldsymbol{\mu}^T(K)]^T$, $\boldsymbol{\mu}(k) = \boldsymbol{\mu}(k-1) + \boldsymbol{\mu}_\varepsilon$, $k \ge 1$, and $\boldsymbol{\mu}(0) = \boldsymbol{\mu}_0$.

The general procedure to calculated system reliability based on the input information listed in Section 4 is summarized in Table A1.

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