Quality-Oriented-Maintenance for Multiple Interactive Tooling Components in Discrete Manufacturing Processes

Yong Chen and Jionghua (Judy) Jin

***Abstract*—In discrete manufacturing processes such as stamping, assembly, or machining processes, product quality, often defined in terms of the dimensional integrity of work pieces, is jointly affected by multiple process variables. During the production phase, the states of tooling components, which are measured by adjustable process variables, are subject to possible random continuous drifts in their means & variances. These drifts of component states may significantly deteriorate product quality during production. Therefore, maintenance of the tooling components with consideration of both their continuous state drifts as well as catastrophic failures is crucial in assuring desired product quality & productivity. In contrast to traditional maintenance models where product quality has not been well addressed, especially for discrete manufacturing processes, a general quality-oriented-maintenance methodology is proposed in this paper to minimize the overall production costs. In this research, the total production cost includes product quality loss due to process drifts, productivity loss due to catastrophic failures, and maintenance costs. The quality-oriented-maintenance model is built based on a response model linking process variables with multidimensional product quality. It can be obtained either from engineering analysis for specific processes, or from statistical design of experiments. Three typical multi-component maintenance models are investigated under the general quality-oriented-maintenance framework. A case study for a sheet-metal stamping process is presented to demonstrate the effectiveness of the proposed methodology.**

***Index Terms*—Discrete manufacturing processes, multi-component systems, preventive maintenance, quality-oriented-maintenance.**

Acronyms[1]

AR	age resetting
MBR	modified block resetting
MTTF	mean time to failure
QOM	quality-oriented-maintenance
SBR	simple block resetting

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[1]The singular and plural of an acronym are always spelled the same.

Notation

$\mathbf{a}$	vector of preventive resetting ages in AR
$\arg\min$	the value of the given for which the value of the given expression attains its minimum value
b	fixed length of the block resetting interval
$c_i^p > 0$	preventive resetting cost
$c_i^f > c_i^p$	unexpected failure cost
c^p	$\equiv \sum_{i=1}^{n} c_i^p$, the summation of the preventive resetting costs of all adjustable process variables
f	function used in a general response model
h	threshold value in MBR
q	coefficient in the quadratic loss function
$Q(\mathbf{s})$	the expected quality loss at age $\mathbf{s}$
s_i	the age of the i^{th} adjustable process variable, i.e., the time since its last resetting
$\mathbf{s}$	$[s_1 \ \ldots \ s_n]^T$, the vector of ages of all adjustable process variables
$\mathbf{V}$	Cov$[\mathbf{z}]$, assumed as a diagonal matrix
$w_i(s_i)$	s-independent random variables characterizing the uncertainty of the drift of X_i, $i = 1, 2, \ldots, n$
X_i	the i^{th} adjustable process variable, $i = 1, \ldots, n$
$\mathbf{X} \in R^n$	vector of adjustable process variables
Y	product quality characteristic
$\mathbf{z}$	$[z_1 \ \ldots \ z_m]^T$, random vector of noise variables with $E[\mathbf{z}] = \mathbf{0}$
$\boldsymbol{\alpha}$	coefficients for the adjustable process variables in the response model
$\boldsymbol{\beta}$	coefficients for the noise variables in the response model
$\boldsymbol{\Gamma}$	coefficients for the interaction effects between the adjustable process variables & the noise variables in the response model
δ_i	mean of X_i at age 0
$\boldsymbol{\delta}$	$[\delta_1 \ \ldots \ \delta_n]^T$
σ_{0i}^2	variance of X_i at age 0
σ_i^2	variance of drifting per unit time for X_i
σ_ε^2	Var$[\varepsilon]$
$\boldsymbol{\theta}_0$	$[\sigma_{01}^2 \ \ldots \ \sigma_{0n}^2]^T$
$\boldsymbol{\theta}$	$[\sigma_1^2 \ \ldots \ \sigma_n^2]^T$
μ_i	mean drifting rate of X_i
$\boldsymbol{\mu}$	$[\mu_1 \ \ldots \ \mu_n]^T$
$\lambda_i \geq 0$	the hazard rate of component i
ε	model error of the response model with $E[\varepsilon] = 0$
η	the baseline constant of the response model

I. Introduction

In manufacturing processes, many process variables that interact in a complicated fashion affect the product quality outputs. For a discrete-part manufacturing process, response models [1] have been widely used to describe the dependency of the product quality measure Y on the process variables ($\mathbf{X}$, $\mathbf{z}$) ass

$$Y = f(\mathbf{X}, \mathbf{z}) \tag{1}$$

where $\mathbf{X}$ is named as the adjustable process variables (process variables which can be adjusted offline), and $\mathbf{z}$ is named as the noise variables (process variables varying randomly, and not adjustable). The relationship in (1) can often be expressed in the format of a regression model. This type of regression model usually does not contain input-output transient dynamics, but it does emphasize the steady-state dependency of system inputs & outputs. For example, stamping processes, panel assembly processes, and machining processes are three typical discrete manufacturing processes in which the product quality is usually defined as the dimensional integrity of workpieces. A tooling component is usually not only subjected to catastrophic failures, but also subjected to wear-out that is reflected by a random drift of an associated process variable. The drift of this process variable from its designed nominal value may significantly deteriorate the product quality. The degraded states of tooling components can be reset to their designed nominal values during the production phase through calibration/readjustment and/or replacing the corresponding tooling components. Thus, the process variable associated with a tooling component is often adjustable through maintenance. Properly maintaining the adjustable process variables associated with tooling components is crucial in assuring product quality, and productivity of a manufacturing process. Most traditional system maintenance policies are focused on the direct downtime or performance loss of each system component. The impact of the state of the system components on the product quality in a manufacturing process is not well addressed in existing maintenance models. In this paper, a quality-oriented-maintenance (QOM) methodology is proposed for discrete manufacturing processes to minimize the overall production costs, including product quality loss, productivity loss, and maintenance costs.

The understanding of the relationship between the process variables and the dimensional deviations of products has been advanced greatly in recent literature. Statistical experimental design & engineering analysis have been widely used to derive quantitative response models in the form of (1) [2]–[7]. The quality cost or loss due to the dimensional deviations is often measured using the quadratic loss function as first proposed by Taguchi [8]:

$$L(Y) = q(Y - \tau)^2,$$

where $L(\cdot)$ represents the loss function, and the constant $q > 0$ is determined by financial considerations. The s-expected loss can be expressed as

$$E\left[L(Y)\right] = q\mathrm{Var}[Y] + q\left(E[Y] - \tau\right)^2 \tag{2}$$

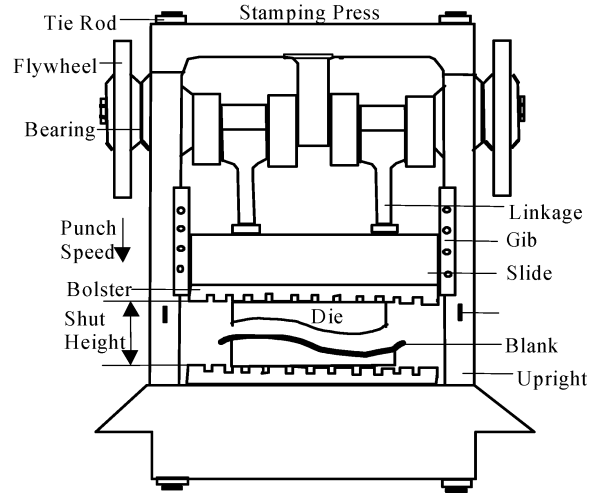

Fig. 1. Sheet metal stamping process.

The common characteristics of the response models and loss functions can be summarized as

1) *multiple* process variables impacting on the product quality; and
2) interaction terms (or product terms) of process variables existing in the quality loss function.

Due to the interaction terms of the process variables, the maintenance schedules of different tooling components cannot be studied separately, which leads to the major challenge of the QOM research. More examples are provided as follows, to illustrate the concept of QOM.

A. Examples of Quality-Oriented-Maintenance Applications

An experimental design was carried out for a sheet metal stamping process shown in Fig. 1 [3]. Both categories, adjustable variables and noise variables, are selected for the experiment. The adjustable process variables include: outer shut-height, inner shut-height, punch speed, and blank washer pressure. The noise variables include: blank thickness, and lubrication. Based on the ANOVA, outer shut-height (X_1), and inner shut-height (X_2) are two significant adjustable process variables, and blank thickness (z_1) is the only significant noise variable. The product quality characteristic (Y) is the dimensional deviation of a critical point on the workpiece from its nominal location. The target value of Y is $\tau = 0$. The variable measurements of outer shut-height, and inner shut-height reflect the worn states of two separate tooling components known as outer die, and inner die, respectively. During production, the wear of the outer die, and the inner die can be characterized as the positive drift of the outer shut-height, and the inner shut height, respectively, from their initial settings. Therefore, the maintenance of the corresponding dies should be scheduled to maintain these two shut heights within a certain region to assure satisfactory product quality. A maintenance decision should be made to achieve minimal total cost, including the maintenance costs, as well as the cumulative loss due to deteriorated product quality.

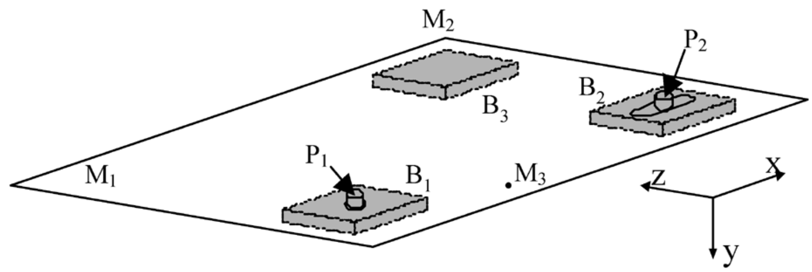

Fig. 2. An example of a panel assembly process.

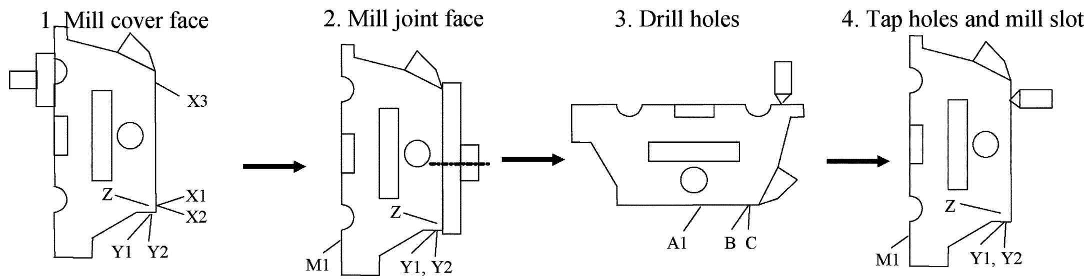

Fig. 3. An example of a machining process.

The response model of this stamping process is obtained in [3] as

$$Y = -0.1136X_1 - 0.0219X_2 + 0.0190z_1 - 0.0130X_1z_1 + \varepsilon, \quad (3)$$

where the variances of ε, and z_1 are obtained as $\sigma_\varepsilon^2 = 0.05$, and $\sigma_z^2 = 0.23$, respectively. All values of the process variables in (3) are coded values, which are certain linear transformations of the corresponding real physical values. From (2) & (3), with $\tau = 0$, the s-expected quality loss for this stamping process is

$$E[L(Y)] = q\left((0.0190 - 0.0130X_1)^2\sigma_z^2\right. \\ \left. + q\left(0.0129X_1^2 + 0.0050X_1X_2 + 0.0005X_2^2\right) + q\sigma_\varepsilon^2.$$

Due to the interaction term $0.0050qX_1X_2$, the expected quality loss function is not separable. As a result, to minimize the cumulative quality loss during production, an optimal maintenance decision for one tooling component is dependent on the maintenance schedules of others. A decision must be made jointly for all tooling components rather than for each single component separately.

QOM is also needed in panel assembly processes, as shown in Fig. 2. In panel assembly processes, locators such as locating pins (e.g., P_1 & P_2 in Fig. 2), and locating blocks (e.g., B_1, B_2, and B_3 in Fig. 2) are used to hold the workpiece at a fixed position. These locators are major tooling components affecting product quality, which are defined as the dimensional deviations of the selected points on the workpiece (e.g., M_1, M_2, M_3 in Fig. 2). The adjustable process variables are the wear of locators. A locator with significant wear can be replaced by a new locator without wear. A response model has been developed in [9] & [5] based on engineering analysis to quantify the relationship between workpiece deviations & locator wear. QOM will periodically replace locators to reduce quality loss due to large dimensional deviations of products.

Another QOM application is an engine head machining process, as shown in Fig. 3. Product quality in machining processes is usually defined as deviations of geometric features in the workpieces. The major tooling components affecting product quality are machining tools such as cutters, drills, and taps, as well as a number of locators. The relationship between workpiece geometric features & machining errors (cutting tools & fixtures) is quantified in [4] through engineering analysis. QOM will periodically replace or recalibrate cutting tools & locators, thus reducing quality loss due to large dimensional deviations of geometric features.

B. Literature Review

From the examples discussed in the previous subsection, the QOM problem typically involves multiple (tooling) components. Multi-component maintenance models are surveyed by Cho & Parlar [10]. Various multi-component maintenance policies have been studied in recent years [11]–[15]. Most of these multi-component maintenance models consider economic dependency among components. It assumed that an opportunity exists for group replacement on several components, provided that a joint replacement cost of several components is less than that of the separate replacements of individual ones. In most existing models, the degradation cost is either ignored, or assigned to each component *separately*. Due to interaction terms of process variables in the defined quality loss function, the cost induced by degradation in the QOM model should be assigned to all adjustable process variables *jointly*. Few multi-component maintenance models in the literature capture the joint degradation cost. Although general degradation costs are addressed in [16], the model in [16] does not consider the component catastrophic failures. Ozekici [17] proposed a very general replacement model with general degradation cost structure. But the *optimal* replacement policy obtained under that general model [17] is complicated, and hard to implement. Important insight has been given in [17] into why the *optimal* policy of multi-component systems is difficult & counterintuitive.

This paper aims to develop a new QOM model to consider the following characteristics:

(1) *joint and interactive* effects of multiple adjustable process variables on product quality loss;
(2) *easy-to-implement* replacement/resetting polices, such as SBR, and MBR; and
(3) two different types of failures due to random drifts, and *catastrophic failures*.

These three characteristics are crucial for effective, practical implementation of QOM. Based on our survey of the literature, no existing maintenance model simultaneously captures these three characteristics, especially for discrete manufacturing processes. In this paper we develop a response-model-based methodology to study QOM. The proposed model is presented in Section II. Various practical multi-component maintenance policies are investigated in Section III. The properties of the maintenance policies are investigated in Section IV. Finally in Section V, one example of the stamping process is analyzed to demonstrate the developed methodology.

II. Multi-Component Quality-Oriented-Maintenance Model

The proposed QOM model consists of three major sub-models describing: (a) drifts of adjustable process variables along with their catastrophic failures; (b) effects of process variables on product quality; and (c) maintenance costs, and product quality loss.

A. Drift and Catastrophic Failure Model for Adjustable Process Variables

Adjustable process variables are measures of the states of functional (tooling) components in a manufacturing process. For example, drill geometry is the measure of drill state in drilling processes; shut height, or open distance of a die, can represent the wear state of the corresponding die in stamping processes; and locator geometry characterizes the state of the locator wear-out. In this paper, we assume that each adjustable process variable is corresponding to one tooling component. The tooling component whose state is measured by the i_{th} adjustable process variable, X_i, is referred to as component i. The drift of an adjustable process variable over time is modeled as

$$X_i(s_i) = X_i(0) + \mu_i s_i + w_i(s_i), \quad s_i \geq 0, \quad i = 1, 2, \ldots, n, \tag{4}$$

where s_i is the age (the time since its last resetting) of the i^{th} process variable, $E[X_i(0)] = \delta_i$, and $\text{Var}[X_i(0)] = \sigma_{0i}^2$; $E[w_i(s_i)] = 0$, and $\text{Var}[w_i(s_i)] = \sigma_i^2 s_i$; δ_i, σ_{0i}^2, μ_i, and σ_i^2 are known constants.

Linear trend modeling is used in (4) for both the mean and variance of X_i. A lot of processes exhibit changes with linear trends [18]–[21]. The variance of $w_i(s_i)$ in (4) can capture the uncertainty of the mean shift parameter, and the random change of the working environment.

Because the drift of an adjustable process variable only depends on its own age, the mean and variance of X_i given $\mathbf{s}$ (the ages of the adjustable process variables) can be obtained as

$$E[X_i(s_i)] = \delta_i + \mu_i s_i \tag{5}$$
$$\text{Var}[X_i(s_i)] = \sigma_{0i}^2 + \sigma_i^2 s_i. \tag{6}$$

We assume that the catastrophic failures are mainly due to some unexpected random events, such as the empty hit, excessive debris, or misadjustment by operators in the manufacturing processes. Because no obvious degradation mechanism is associated with these events, their occurrences are often with constant rates. Therefore, the catastrophic failure of each component in the system is assumed to have a constant hazard rate. That is, the catastrophic failure time follows an exponential distribution

$$\Pr\{\text{Component } i \text{ fails before age } s_i\} = 1 - e^{-\lambda_i s_i}.$$

B. Response Model

The study of the impacts of the adjustable process variables on product quality is based on the response model

$$Y(\mathbf{s}) = \eta + \boldsymbol{\alpha}^T \mathbf{X}(\mathbf{s}) + \boldsymbol{\beta}^T \mathbf{z} + \mathbf{X}(\mathbf{s})^T \boldsymbol{\Gamma} \mathbf{z} + \varepsilon, \tag{7}$$

where $\mathbf{X}(\mathbf{s})$ contains the adjustable process variables, i.e., $\mathbf{X}(\mathbf{s}) \equiv [X_1(\mathbf{s}) \ \ldots \ X_n(\mathbf{s})]^T$; $Y(\mathbf{s})$ denotes the dependency of the quality characteristic Y on $\mathbf{s}$; and $\eta \in R$, $\boldsymbol{\alpha} \in R^n$, $\boldsymbol{\beta} \in R^m$, and $\boldsymbol{\Gamma} \in R^{n \times m}$ are parameters in the model. The parameters in (7) can be either fitted based on the results from statistical experimental design, or obtained through engineering analysis based on the process & product design information. In the literature of design of experiments, the terms $\boldsymbol{\alpha}^T \mathbf{X}(\mathbf{s})$, $\boldsymbol{\beta}^T \mathbf{z}$, and $\mathbf{X}(\mathbf{s})^T \boldsymbol{\Gamma} \mathbf{z}$ shown in model (7) are called main effects of adjustable process variables, main effects of noise variables, and interaction effects between adjustable process variables & noise variables, respectively. These three effects are well accepted as the most significant effects in many quality engineering applications. In this paper, we restrict our study on models in which these three effects are much more s-significant than the others. A discussion will be presented in Section III-D for models with other higher order effects.

Based on (7), given $\mathbf{X}(\mathbf{s})$, the s-expectation, and variance of $Y(\mathbf{s})$ can be obtained from (7) as

$$E(Y(\mathbf{s})|\mathbf{X}(\mathbf{s})) = \eta + \boldsymbol{\alpha}^T \mathbf{X}(\mathbf{s})$$
$$Var(Y(\mathbf{s})|\mathbf{X}(\mathbf{s})) = \left(\boldsymbol{\beta}^T + \mathbf{X}(\mathbf{s})^T \boldsymbol{\Gamma}\right) \mathbf{V} \left(\boldsymbol{\beta} + \boldsymbol{\Gamma}^T \mathbf{X}(\mathbf{s})\right) + \sigma_\varepsilon^2.$$

C. Product Quality Loss, and Maintenance Costs

Using the quadratic loss function, the s-expected quality loss given $\mathbf{X}(\mathbf{s})$ can be written as

$$\begin{aligned} &E[L(Y(\mathbf{s}))|\mathbf{X}(\mathbf{s})] \\ &= E\left[q(Y(\mathbf{s}) - \tau)^2 |\mathbf{X}(\mathbf{s})\right] \\ &= q\text{Var}[Y(\mathbf{s})|\mathbf{X}(\mathbf{s})] + q(E[Y(\mathbf{s})|\mathbf{X}(\mathbf{s})] - \tau)^2 \\ &= q\Big(\mathbf{X}(\mathbf{s})^T(\boldsymbol{\Gamma}\mathbf{V}\boldsymbol{\Gamma}^T + \boldsymbol{\alpha}\boldsymbol{\alpha}^T)\mathbf{X}(\mathbf{s}) \\ &\quad + 2\left(\beta^T \mathbf{V}\boldsymbol{\Gamma}^T + (\eta - \tau)\boldsymbol{\alpha}^T\right)\mathbf{X}(\mathbf{s}) \\ &\quad + \boldsymbol{\beta}^T \mathbf{V}\boldsymbol{\beta} + (\eta - \tau)^2 + \sigma_\varepsilon^2\Big) \end{aligned} \tag{8}$$

In (8), the term $q\mathbf{X}(\mathbf{s})^T(\mathbf{\Gamma V \Gamma}^T + \boldsymbol{\alpha\alpha}^T)\mathbf{X}(\mathbf{s})$ represents the interactive impacts of the adjustable process variables on the product quality loss. Given $\mathbf{s}$, the ages for the adjustable process variables, $\mathbf{X}(\mathbf{s})$ is still random due to the uncertainty of their drifting processes. Taking the s-expectation on $\mathbf{X}(\mathbf{s})$,

$$\begin{aligned} Q(\mathbf{s}) &= \underset{\mathbf{X}(\mathbf{s})}{E}\left[\underset{\mathbf{z}}{E}\left[L\left(Y(\mathbf{s})\right)|\mathbf{X}(\mathbf{s})\right]\right] \\ &= q\underset{\mathbf{X}(\mathbf{s})}{E}\Big[\mathbf{X}(\mathbf{s})^T(\mathbf{\Gamma V\Gamma}^T+\boldsymbol{\alpha\alpha}^T)\mathbf{X}(\mathbf{s}) \\ &\qquad + 2\left(\boldsymbol{\beta}^T\mathbf{V\Gamma}^T+(\eta-\tau)\boldsymbol{\alpha}^T\right)\mathbf{X}(\mathbf{s}) \\ &\qquad + \boldsymbol{\beta}^T\mathbf{V}\boldsymbol{\beta}+(\eta-\tau)^2+\sigma_\varepsilon^2\Big] \\ &= q\underset{\mathbf{X}(\mathbf{s})}{E}\left[\mathbf{X}(\mathbf{s})^T\mathbf{B}\mathbf{X}(\mathbf{s})+\mathbf{p}^T\mathbf{X}(\mathbf{s})+d\right] \\ &= q\Big(E\left[\mathbf{X}(\mathbf{s})\right]^T\mathbf{B}E\left[\mathbf{X}(\mathbf{s})\right]+\mathbf{p}^TE\left[\mathbf{X}(\mathbf{s})\right] \\ &\qquad + \sum_{i=1}^n b_{ii}\mathrm{Var}\left[X_i(\mathbf{s})\right]+d\Big) \end{aligned} \tag{9}$$

where $\mathbf{B}=(b_{ij})_{n\times n}\equiv\mathbf{\Gamma V\Gamma}^T+\boldsymbol{\alpha\alpha}^T$, $\mathbf{p}^T\equiv 2(\boldsymbol{\beta}^T\mathbf{V\Gamma}^T+(\eta-\tau)\boldsymbol{\alpha}^T)$, and $d\equiv\boldsymbol{\beta}^T\mathbf{V}\boldsymbol{\beta}+(\eta-\tau)^2+\sigma_\varepsilon^2$.

Define $\mathbf{b}\equiv[b_{11}\ \ldots\ b_{nn}]^T$. Substituting (5) & (6) into (9), we have

$$Q(\mathbf{s})=q\left(\mathbf{s}^T(\mathbf{B}^*)\mathbf{s}+(\mathbf{p}^*)^T\mathbf{s}+d^*\right) \tag{10}$$

where $\mathbf{B}^*\equiv(\boldsymbol{\mu\mu}^T)\circ\mathbf{B}$, $\mathbf{p}^*\equiv\boldsymbol{\mu}\circ\mathbf{p}+2(\mathbf{B}\circ\boldsymbol{\mu})^T\boldsymbol{\delta}+\mathbf{b}\circ\boldsymbol{\theta}$, and $d^*\equiv d+\mathbf{p}^T\boldsymbol{\delta}+\boldsymbol{\delta}^T\mathbf{B}\boldsymbol{\delta}+\mathbf{b}^T\boldsymbol{\theta}_0$. The product $\circ$ is an extension of the Hadamard product, which is defined as follows.

Definition 1: For two $n_1\times n_2$ matrices of $\mathbf{C}=(c_{ij})$, and $\mathbf{D}=(d_{ij})$, and one $n_2\times 1$ vector of $\mathbf{u}=(u_i)$, the Hadamard product of $\mathbf{C}$ & $\mathbf{D}$ results in a $n_1\times n_2$ matrix of the element-wise product

$$\mathbf{C}\circ\mathbf{D}=(c_{ij}d_{ij})_{n_1\times n_2}$$

Further, $\mathbf{C}\circ\mathbf{u}$ is defined as an $n_1\times n_2$ matrix as

$$\mathbf{C}\circ\mathbf{u}=(c_{ij}u_j)_{n_1\times n_2}.$$

Note that $\mathbf{C}$ and $\mathbf{u}$ have different dimensions, while the Hadamard product is usually defined only for two matrices of the same dimension. Therefore, the operator defined in Definition 1 is a generalized Hadamard product.

Let $\mathbf{s}(t)$ denote the ages of the adjustable process variables at time $t\in[0,\infty)$. The ages of the adjustable process variables $s_i(t), i=1,\ldots,n$ are s-independent random variables because catastrophic failures of each tooling component are s-independent. Taking the s-expectation of (10) on $\mathbf{s}(t)$,

$$\begin{aligned} E\left[Q\left(\mathbf{s}(t)\right)\right] &= qE\left[\mathbf{s}(t)^T(\mathbf{B}^*)\mathbf{s}(t)+(\mathbf{p}^*)^T\mathbf{s}(t)+d^*\right] \\ &= q\Big(E\left[\mathbf{s}(t)\right]^T(\mathbf{B}^*)E\left[\mathbf{s}(t)\right]+(\mathbf{p}^*)^TE\left[\mathbf{s}(t)\right] \\ &\qquad + \sum_{i=1}^n b_{ii}^*\mathrm{Var}\left[s_i(t)\right]+d^*\Big) \end{aligned} \tag{11}$$

where b_{ii}^* is the $(i,i)^{\text{th}}$ element of $\mathbf{B}^*$. The following lemma shows properties of $\mathbf{B}^*$, and b_{ii}^*.

Lemma 1: $\mathbf{B}^*$ is nonnegative definite, and $b_{ii}^*\geq 0$, $\forall i=1,\ldots,n$.

Lemma 1 is not difficult to see from the definitions of $\mathbf{B}^*$, and b_{ii}^*. In addition to the quality loss, maintenance actions are subject to maintenance costs. If the i^{th} adjustable process variable is reset preventively at a scheduled time, we assume that a cost c_i^p is suffered. If it is reset at the unexpected catastrophic failure of its corresponding tooling component, we assume that a cost c_i^f is suffered.

III. Multi-Component Maintenance Policies

The drifted adjustable process variables need to be reset during the production phase by readjustment or replacement of the corresponding tooling component. Maintenance decisions will determine when each of the adjustable process variables should be reset. Depending on the requirements on feasibility, flexibility, and cost efficiency of the maintenance actions, three replacement/resetting policies are widely studied for multi-component systems. Each of them will be investigated in this section. Because the resetting of the adjustable process variables is not necessarily accomplished by component replacement, we will use resetting instead of replacing when we refer to a maintenance policy in the literature.

A. Simple Block Resetting (Replacement)

Under the SBR policy, each adjustable process variable is reset when its corresponding tooling component has a catastrophic failure. Additionally, regardless of their individual ages, all adjustable process variables are preventively reset at times kb $(k=1,2,\ldots)$. The optimal maintenance decision aims to minimize the expected long-run average cost by choosing the optimal b. The s-expected long-run average cost under the SBR policy is given in the following lemma.

Lemma 2: The s-expected long-run average cost under the SBR policy is

$$\Phi(b)=\sum_{i=1}^n c_i^f\lambda_i+\frac{c^p+\int_0^b E\left[Q\left(\mathbf{s}(t)\right)\right]dt}{b}. \tag{12}$$

Please refer to Appendix I for the proof of Lemma 2. From Lemma 2 & (11), $E[Q(\mathbf{s}(t))]$ under the SBR policy, i.e. $\Phi(b)$, can be calculated if $E[s_i(t)]$ & $\mathrm{Var}[s_i(t)]$ are known, which are given in the Lemma 3.

Lemma 3: Under the SBR policy, for $t\in(0,b)$,

$$\begin{aligned} E\left[s_i(t)\right] &= \frac{1}{\lambda_i}(1-e^{-\lambda_i t}), \quad \text{and} \\ \mathrm{Var}\left[s_i(t)\right] &= \frac{1-2\lambda_i te^{-\lambda_i t}-e^{-2\lambda_i t}}{\lambda_i^2} \end{aligned}$$

Proof: Because the Poisson process has stationary & s-independent increments,

$$\begin{aligned} \Pr\left\{s_i(t)>x\right\} &= \Pr\left\{\text{No failure during } (t-x,t)\right\} \\ &= \Pr\left\{\text{No failure during } (0,x)\right\} \\ &= e^{-\lambda_i x}, \qquad \text{for } 0<x<t \end{aligned} \tag{13}$$

Also, $s_i(t)$ is nonnegative. Therefore,

$$E[s_i(t)] = \int_0^t e^{-\lambda_i x} dx = \frac{1}{\lambda_i}(1 - e^{-\lambda_i t}),$$

$$E[s_i^2(t)] = \int_0^t 2x e^{-\lambda_i x} dx = \frac{2(1 - e^{-\lambda_i t} - \lambda_i t e^{-\lambda_i t})}{\lambda_i^2},$$

and

$$\mathrm{Var}[s_i(t)] = E[s_i^2(t)] - (E[s_i(t)])^2 = \frac{1 - 2\lambda_i t e^{-\lambda_i t} - e^{-2\lambda_i t}}{\lambda_i^2}. \qquad \square$$

From Lemma 1, it can be seen that the optimal value of b does not depend on the failure resetting cost c_i^f. Based on Lemma 2 & Lemma 2, we have Result 1.

Result 1: The optimal SBR policy can be obtained by solving the following nonlinear optimization problem with one decision variable:

$$b^* = \arg\min_b \Phi(b) = \arg\min_b \frac{c^p + \int_0^b E[Q(\mathbf{s}(t))]\,dt}{b} \quad \text{subject to } b > 0, \tag{14}$$

where $E[Q(\mathbf{s}(t))]$ is calculated by substituting $E[s_i(t)]$ & $\mathrm{Var}[s_i(t)]$ in (11) with Lemma 3. By calculating the first order derivative, if $b^* < \infty$, it should satisfy

$$\int_0^{b^*} (E[Q(\mathbf{s}(b^*))] - E[Q(\mathbf{s}(t))])\,dt = c^p. \tag{15}$$

Corollary 1: If $b^* < \infty$ is the optimal SBR interval, then

$$\Phi(b^*) = \sum_{i=1}^n c_i^f \lambda_i + E[Q(\mathbf{s}(b^*))].$$

Proof: From (15),

$$\int_0^{b^*} E[Q(\mathbf{s}(t))]\,dt = b^* E[Q(\mathbf{s}(b^*)] - c^p.$$

Using Lemma 2,

$$\Phi(b^*) = \sum_{i=1}^n c_i^f \lambda_i + \frac{c^p + \int_0^{b^*} E[Q(\mathbf{s}(t))]\,dt}{b^*} = \sum_{i=1}^n c_i^f \lambda_i + E[Q(\mathbf{s}(b^*))]. \qquad \square$$

B. Modified Block Resetting

Although the SBR policy is simple to understand and implement, it may result in the resetting of newly adjusted process variables. To avoid this unnecessary waste, the MBR policy is proposed by Berg & Epstein [22]. Under the MBR policy, an adjustable process variable is reset based on two occurrences: component catastrophic failure occurrence, and pre-scheduled block resetting times ($kb, k = 1, 2, \ldots$) occurrence if the ages of the adjustable process variables are not less than $h \in [[0, b]]$ at a pre-scheduled block resetting time. Let $s_i(b)$ denote the random age of tooling component i at the end of a block resetting interval. The s-expected long-run average cost is

$$\Phi(b,h) = \frac{E(\text{cost during one preventive resetting interval})}{b} = \sum_{i=1}^n c_i^f \lambda_i + \frac{\sum_{i=1}^n c_i^p e^{-\lambda_i h} + \int_0^b E[Q(\mathbf{s}(t))]\,dt}{b} \tag{16}$$

One difference between (16) and Lemma 2 is in the preventive resetting cost. Under an MBR policy, only adjustable process variables with ages no less than h are preventively reset at the end of a block resetting interval. As a result, the preventive resetting cost under the MBR policy is

$$\sum_{i=1}^n c_i^p \Pr\{s_i(b) \geq h\} = \sum_{i=1}^n c_i^p e^{-\lambda_i h},$$

where $\Pr\{s_i(b) \geq h\}$ is calculated from (13).

For the MBR policy, (11) can still be used to calculate $E[Q(\mathbf{s}(t))]$. However, the calculation of $E[s_i(t)]$ & $E[s_i^2(t)]$ is different from the SBR policy due to the possible nonzero $s_i(0)$, the age of adjustable process variable i at the beginning of a block resetting interval. The following lemma evaluates $E[s_i(t)]$, and $\mathrm{Var}[s_i(t)]$, for the MBR policy.

Lemma 4: Under the MBR policy (b, h), for $t \in (0, b)$

$$E[s_i(t)] = \frac{1}{\lambda_i} - \left(\frac{1}{\lambda_i} + h\right) e^{-\lambda_i(h+t)}$$

$$Var[s_i(t)] = \frac{1}{\lambda_i^2} - \frac{h^2\lambda_i + 2(h\lambda_i + 1)t}{\lambda_i} e^{-(h+t)\lambda_i} - \frac{(1 + h\lambda_i)^2}{\lambda_i^2} e^{-2(h+t)\lambda_i}$$

Proof: Conditioning on the last catastrophic failure time of component i during (0, t),

$$\begin{aligned} E[s_i(t)] &= \Pr\{\text{no failure on } (0,t))\} \cdot E[s_i(t)|\text{ no failure on } (0,t)] \\ &\quad + \int_0^t E[s_i(t)|\text{ a failure at time } t-x] \times \lambda_i e^{-\lambda_i x} dx \\ &= e^{-\lambda_i t} E[s_i(0) + t] + \int_0^t x \lambda_i e^{-\lambda_i x} dx \\ &= \frac{1}{\lambda_i} - \left(\frac{1}{\lambda_i} + h\right) e^{-\lambda_i(h+t)} \end{aligned}$$

$$\begin{aligned}
E\left[s_i^2(t)\right] &= \Pr\{\text{no failure on}(0,t))\} \\
&\quad \cdot E\left[s_i^2(t)\right| \text{no failure on } (0,t)] \\
&\quad + \int_0^t E\left[s_i^2(t)\right| \text{a failure at time } t-x] \\
&\quad \times \lambda_i e^{-\lambda_i x} dx \\
&= e^{-\lambda_i t} E[(s_i(0)+t)^2] + \int_0^t x^2 \lambda_i e^{-\lambda_i x} dx \\
&= \frac{1}{\lambda_i^2}\Big(2 - e^{-(h+t)\lambda_i} \\
&\qquad \times \left(2 + 2\lambda_i t + h^2\lambda_i^2 + 2h\lambda_i\left(1+\lambda_i t\right)\right)\Big) \\
\mathrm{Var}\left[s_i(t)\right] &= E\left[s_i^2(t)\right] - \left(E\left[s_i(t)\right]\right)^2 \\
&= \frac{1}{\lambda_i^2} - \frac{h^2\lambda_i + 2(h\lambda_i+1)t}{\lambda_i} e^{-(h+t)\lambda_i} \\
&\quad - \frac{(1+h\lambda_i)^2}{\lambda_i^2} e^{-2(h+t)\lambda_i} \qquad \square
\end{aligned}$$

Result 2: The optimal MBR policy can be obtained by solving the following nonlinear optimization problem with two decision variables:s

$$\begin{aligned}
(b^*, h^*) &= \arg\min_{b,h} \Phi(b,h) \\
&= \arg\min_{b,h} \frac{\sum_{i=1}^n c_i^p e^{-\lambda_i h} + \int_0^b E\left[Q\left(\mathbf{s}(t)\right)\right] dt}{b} \\
&\quad \text{subject to } b > 0, \ \text{ and } \ 0 \le h \le b,
\end{aligned}$$

where $E[Q(\mathbf{s}(t))]$ is calculated by substituting $E[s_i(t)]$ & $\mathrm{Var}[s_i(t)]$ in (11) with Lemma 4. Let b_h denote the optimal value of $b \ge h$ which minimizes $\Phi(b,h)$ under fixed h. Similar to the SBR policy, if $b_h < \infty$, then

$$\int_0^{b_h} \left(E\left[Q\left(\mathbf{s}(b_h)\right)\right] - E\left[Q\left(\mathbf{s}(t)\right)\right]\right) dt = \sum_{i=1}^n c_i^p e^{-\lambda_i h},$$

and

$$\Phi(b_h, h) = \sum_{i=1}^n c_i^f \lambda_i + E\left[Q\left(\mathbf{s}(b_h)\right)\right].$$

The properties of b_h above will be used in Section IV.

C. Age Resetting

For both the SBR policy and the MBR policy, all adjustable process variables share the same maintenance policy (b or (b, h)). Special drifting and/or failure characteristics of individual adjustable process variables are not fully considered. Under an AR policy, each adjustable process variable has its own resetting schedule. The i^{th} adjustable process variable is reset when catastrophic failures of the i^{th} tooling component occur. In addition, it is preventively reset whenever its age reaches a_i. Therefore, an AR policy is determined by $\mathbf{a} \equiv [a_1 \ \ldots \ a_n]^T$. The drawback of the AR policy is that preventive maintenance cannot be planned in advance, and tracking the ages of adjustable process variables involves significant administration. The s-expected long-run average cost under an AR policy is given in the following lemma.

Lemma 5: Under the AR policy $\mathbf{a}$, the s-expected long-run average cost is

$$\Phi(\mathbf{a}) = \sum_{i=1}^n \lambda_i \left(c_i^f - c_i^p + \frac{c_i^p}{1 - e^{-\lambda_i a_i}}\right) + \lim_{t\to\infty} E\left[Q\left(\mathbf{s}(t)\right)\right]$$

Proof: Let $\Phi_m(\mathbf{a})$ denote the s-expected long-run average maintenance cost, and $\Phi_q(\mathbf{a})$ denote the s-expected long-run average quality loss,

$$\Phi(\mathbf{a}) = \Phi_m(\mathbf{a}) + \Phi_q(\mathbf{a})$$

The catastrophic failure & its induced cost for each tooling component can be considered as a renewal process to restore the failed component at each resetting time. Based on the result for long-run average cost or return for renewal reward processes, when the catastrophic failure of each component has a constant hazard rate, the s-expected long-run average maintenance cost is

$$\begin{aligned}
\Phi_m(\mathbf{a}) &= \sum_{i=1}^n \frac{c_i^p e^{-\lambda_i a_i} + c_i^f (1 - e^{-\lambda_i a_i})}{\int_0^{a_i} e^{-\lambda_i t} dt} \\
&= \sum_{i=1}^n \lambda_i \left(c_i^f - c_i^p + \frac{c_i^p}{1 - e^{-\lambda_i a_i}}\right)
\end{aligned}$$

For the long-run average quality loss,

$$\Phi_q(a) = \lim_{T\to\infty} \frac{\int_0^T E\left[Q\left(\mathbf{s}(t)\right)\right]}{T} = \lim_{t\to\infty} E\left[Q\left(\mathbf{s}(t)\right)\right]. \quad \square$$

From (11), $\lim_{t\to\infty} E[Q(\mathbf{s}(t))]$ can be obtained based on $\lim_{t\to\infty} E[s_i(t)]$ & $\lim_{t\to\infty} \mathrm{Var}[s_i(t)]$, which are given in the following lemma.

Lemma 6: For an AR policy, we have the following results:

$$\begin{aligned}
\lim_{t\to\infty} E[s_i(t)] &= \frac{1}{\lambda_i} - \frac{a_i e^{-\lambda_i a_i}}{1 - e^{-\lambda_i a_i}} \\
\lim_{t\to\infty} \mathrm{Var}\left[s_i(t)\right] &= \frac{1 + e^{-2a_i\lambda_i} - e^{-a_i\lambda_i}\left(2 + a_i^2\lambda_i^2\right)}{\lambda_i^2 (1 - e^{-a_i\lambda_i})^2}
\end{aligned}$$

Proof: For each adjustable process variable, a renewal occurs at the time of resetting. Let U_i denote the random length of the renewal/resetting interval of component i. Based on the limiting distribution of the age of a renewal process given in Ross [23] (page 116), it is not difficult to see that

$$\begin{aligned}
\lim_{t\to\infty} E\left[s_i(t)\right] &= \frac{E\left[U_i^2\right]}{2E[U_i]} = \frac{1}{\lambda_i} - \frac{a_i e^{-\lambda_i a_i}}{1 - e^{-\lambda_i a_i}} \\
\lim_{t\to\infty} E\left[s_i^2(t)\right] &= \frac{E\left(U_i^3\right)}{3E(U_i)} = \frac{2}{\lambda_i^2} - \frac{a_i e^{-\lambda_i a_i}(2 + a_i\lambda_i)}{(1 - e^{-\lambda_i a_i})\lambda_i}
\end{aligned}$$

The lemma follows from $\lim_{t\to\infty} \mathrm{Var}[s_i(t)] = \lim_{t\to\infty} E[s_i^2(t)] - (\lim_{t\to\infty} E[s_i(t)])^2$. $\square$

Result 3: The optimal AR policy can be obtained by solving the following nonlinear optimization problem with n decision variables:

$$\begin{aligned}
\mathbf{a}^* &= \arg\min_{\mathbf{a}} \Phi(\mathbf{a}) \\
&= \arg\min_{\mathbf{a}} \left(\sum_{i=1}^n \frac{\lambda_i c_i^p}{1 - e^{-\lambda_i a_i}} + \lim_{t\to\infty} E\left[Q\left(\mathbf{s}(t)\right)\right]\right) \\
&\quad \text{subject to } a_i > 0, \qquad i = 1, 2, \ldots, n,
\end{aligned}$$

where $\lim_{t\to\infty} E[Q(\mathbf{s}(t))]$ is calculated by using (11) & Lemma 6.

Although the AR policy usually results in lower cost than the SBR and MBR policies, its implementation is difficult because it applies different maintenance schedules for different system components. However, it is still worthy to study the AR policy, because the optimal cost achieved by an AR policy can be used as a benchmark to assess if the performance of those practical policies, such as SBR, and MBR, are far away from the optimal AR solution. If the cost of the SBR or MBR is close to that of the AR, then we are confident that the efficiency of the SBR or MBR policy is satisfactory.

D. Discussion of Higher Order Effects in Response Model

If higher order effects exist in the response model (7), similar procedures as shown in Sections II & III can still be followed to formulate the maintenance problem as a nonlinear optimization problem. For example, second order effects & interactions could exist for $\mathbf{X}(\mathbf{s})$, that is, there could be an additional term $\mathbf{X}(\mathbf{s})^T\mathbf{A}\mathbf{X}(\mathbf{s})$ in (7) where $\mathbf{A}$ is a matrix of appropriate dimension. Moments of $\mathbf{X}(\mathbf{s})$ with an order higher than two are needed to get $Q(\mathbf{s})$ as in (10). It can be seen that the main results of Section III, i.e., Results 1, 2, and 3, are valid for response models with any order of effects & interactions. However, if the response models were different, then the evaluation of $E[Q(\mathbf{s}(t))]$ would be different. For response models with higher order effects, the calculation of $E[Q(\mathbf{s}(t))]$, as in (11), requires moments of higher order for $\mathbf{s}(t)$, which can be obtained following similar procedures as in Lemmas 3, 4, and 6 for the SBR, MBR, and AR policies, respectively.

IV. Further Discussion on Solutions of the Optimization Problems

In general, studying the properties of an optimal maintenance policy is very difficult. Practical assumptions are needed to simplify the problem. For example, in the literature of traditional preventive replacement problems, assumptions such as monotone increasing hazard rates are made to derive properties of the optimal solutions. In this paper, we assume that $Q(\mathbf{s})$, the loss function corresponding to the product quality produced by tooling components of age $\mathbf{s}$, is a continuous, monotone increasing function of $\mathbf{s}$. This assumption can be interpreted as that a system with more degraded tooling components will produce products with inferior quality. Based on this assumption, the optimal solutions for the SBR & MBR policies are further investigated. The following lemmas will be used in the discussion.

Lemma 7: For the SBR policy, $E[s_i(t)]$ & $Var[s_i(t)]$ are strictly increasing functions of t. For the MBR policy when $t \geq h$, $E[s_i(t)]$ is a strictly increasing function of t, and $Var[s_i(t)]$ is a strictly increasing function of t. For the MBR policy, both $E[s_i(t)]$ & $Var[s_i(t)]$ are increasing functions of h.

Proof: The proof is based on evaluating the derivatives of $E[s_i(t)]$ & $Var[s_i(t)]$ repeatedly. Please refer to Appendix II for the detailed proof.

Lemma 8: If $Q(\mathbf{s})$ is a continuous & increasing function of $\mathbf{s}$, for the SBR policy, $E[Q(\mathbf{s}(t))]$ is a continuous & increasing function of t. For the MBR policy, $E[Q(\mathbf{s}(t))]$ is a continuous & increasing function of t for $t \geq h$. If $Q(\mathbf{s})$ is strictly increasing, so is $E[Q(\mathbf{s}(t))]$.

Proof: Because $Q(\mathbf{s})$ is an increasing function of $\mathbf{s}$, from (10), $\mathbf{s}^T(\mathbf{B}^*)\mathbf{s} + (\mathbf{p}^*)^T\mathbf{s}$ is increasing on $\mathbf{s}$. Further, from Lemma 7, $E[s_i(t)]$ is an increasing function of t. So, $E[\mathbf{s}(t)]^T(\mathbf{B}^*)E[\mathbf{s}(t)] + (\mathbf{p}^*)^T E[\mathbf{s}(t)]$ is an increasing function of t. From Lemmas 1 & 7, $b_{ii}^* \geq 0$, and $Var[s_i(t)]$ is an increasing function of t (if $t \geq h$ for the modified resetting policy). Therefore, $E[\mathbf{s}(t)]^T(\mathbf{B}^*)E[\mathbf{s}(t)] + (\mathbf{p}^*)^T E[\mathbf{s}(t)] + \sum_{i=1}^n b_{ii}^* \mathrm{Var}[s_i(t)]$ is an increasing function of t. Based on (11), this shows that $E[Q(\mathbf{s}(t))]$ is an increasing function of t (if $t \geq h$ for the modified resetting policy). If $Q(\mathbf{s})$ is strictly increasing, obviously $E[Q(\mathbf{s}(t))]$ is also strictly increasing because $E[s_i(t)]$ is strictly increasing. □

A. Optimal Solutions of SBR Policy

The following result shows the property of the optimal solutions for the SBR policy. These properties can be used to facilitate the nonlinear optimization procedure.

Result 4: For the SBR policy, if $Q(\mathbf{s})$ is a continuous & monotone increasing function of $\mathbf{s}$, we have the following:

(i) Any local minimum of (14) is also a global minimum.

(ii) If $\int_0^\infty (E[Q(\mathbf{s}(\infty))] - E[Q(\mathbf{s}(t))])dt \leq c^p$, infinite is an optimal solution to (14); i.e., resetting at only catastrophic failures is an optimal policy. If $\int_0^\infty (E[Q(\mathbf{s}(\infty))] - E[Q(\mathbf{s}(t))])dt > c^p$, a finite optimal solution to (14) exists.

(iii) If $Q(\mathbf{s})$ is strictly increasing, the optimal solution is unique.

Proof: From Lemma 8, when $Q(\mathbf{s})$ is increasing, $E[Q(\mathbf{s}(t)]$ is also increasing.

So, for $b_2 > b_1 \geq 0$, $\int_0^{b_2} (E[Q(\mathbf{s}(b_2))] - E[Q(\mathbf{s}(t))])dt \geq \int_0^{b_1} (E[Q(\mathbf{s}(b_2))] - E[Q(\mathbf{s}(t))])dt \geq \int_0^{b_1} (E[Q(\mathbf{s}(b_1))] - E[Q(\mathbf{s}(t))])dt$. therefore, $\int_0^{b} (E[Q(\mathbf{s}(b))] - E[Q(\mathbf{s}(t))])dt$ is increasing as a function of b.

Then, it results in (i).

To prove (ii), from Result 1, and $\int_0^\infty (E[Q(\mathbf{s}(\infty))] - E[Q(\mathbf{s}(t))])dt \leq c^p$, $d\Phi(b)/db \leq 0, \forall b < \infty$.

Therefore, the optimal solution to (14) equals infinity.

If $\int_0^\infty (E[Q(\mathbf{s}(\infty))] - E[Q(\mathbf{s}(t))])dt > c^p$, because $E[Q(\mathbf{s}(t))]$ is continuous, there must exist $b < \infty$, such that $\int_0^{b} (E[Q(\mathbf{s}(b))] - E[Q(\mathbf{s}(t))])dt = c^p$.

From (i), b is a finite optimal solution taken as the global optimum.

If $Q(\mathbf{s})$ is strictly increasing, so is $E[Q(\mathbf{s}(t))]$. Additionally, $\int_0^{b} (E[Q(\mathbf{s}(b))] - E[Q(\mathbf{s}(t))])dt$ is strictly increasing. So, $\int_0^{b} (E[Q(\mathbf{s}(b))] - E[Q(\mathbf{s}(t))])dt = c^p$ has a unique solution, if any. Thus, the optimal solution to (14) is unique (can be infinite), and (iii) is proved. □

B. Optimal Solutions of Modified Block Resetting Policy

There are two decision variables in MBR policies. Therefore, the study of the properties of the optimal MBR policy is more difficult than that of the SBR policy. The following result and its corollaries show the properties of the optimal MBR policy.

TABLE I
CODING OF ADJUSTABLE PROCESS VARIABLES

Variable	X_1	X_2
Name	Outer Shut-Height (in.)	Inner Shut-Height (in.)
Low (-1)	83.6553	95.9435
High (+1)	83.6725	95.9646

Result 5: For the MBR policy, if $Q(\mathbf{s})$ is strictly increasing, $h_1 \le h_2$, and $b_{h1} < b_{h2}$, then $\Phi(b_{h1}, h_1) < \Phi(b_{h2}, h_2)$.

Proof: When $b_{h1} < b_{h2} < \infty$, and $h_1 \le h_2$, from Lemma 8, $E[Q(\mathbf{s}(b_{h1}))] < E[Q(\mathbf{s}(b_{h2}))]$. Further, from Result 2, $\Phi(b_{h1}, h_1) = \sum_{i=1}^n c_i^f \lambda_i + E[Q(\mathbf{s}(b_{h1}))] < \sum_{i=1}^n c_i^f \lambda_i + E[Q(\mathbf{s}(b_{h2}))] = \Phi(b_{h2}, h_2)$. If $b_{h2} = \infty$, it is easy to see that $\sum_{i=1}^n c_i^f \lambda_i + E[Q(\mathbf{s}(b_{h2}))] \le \Phi(b_{h2}, h_2)$, and the result still follows. □

Corollary 2: For the MBR policy, if $Q(\mathbf{s})$ is strictly increasing, and (b^*, h^*) is a global optimal policy, $h \le h^*$ implies $b_h \ge b^*$.

Proof: We have $b^* = b_{h^*}$. If $h \le h^*$, suppose $b_h < b^*$, from Result 5, $\Phi(b_h, h) < \Phi(b_{h^*}, h^*)$, which contradicts with the definition that (b^*, h^*) is a global optimum. Therefore, $b_h \ge b^*$ if $h \le h^*$. □

In the next corollary, a sufficient condition is given for the MBR policy to have a finite optimal solution.

Corollary 3: If $Q(\mathbf{s})$ is strictly increasing, and there exists a finite optimal SBR policy, a finite optimal MBR policy must also exist.

Proof: The SBR policy can be considered as an MBR policy with $h = 0$. Therefore, the optimal SBR policy is $b_{h=0} < \infty$. From Corollary 2, because $h^* \ge 0$, the global optimal MBR policy (b^*, h^*) must satisfy $b^* \le b_{h=0} < \infty$. □

V. CASE STUDY

A case study is conducted for the stamping process introduced in Section I. The significant adjustable process variables for the stamping process are the outer shut-height (X_1), and inner shut-height (X_2). The response model of this process is given in (3). The coding & its corresponding real values for the adjustable process variables used in the experiment are listed in Table I. Due to the wear-out of outer die & inner die, both the outer shut-height & the inner shut-height can be observed with positive mean shifts. As a result, these dies should be preventively maintained to reset shut heights, and maintain product quality. In addition, catastrophic failures occur on the corresponding dies of the outer & inner shut-heights. Once a catastrophic failure occurs, the die should be repaired, and the shut-height is reset at the same time. The model parameters used in the maintenance model of Section II are listed in Table II.

The parameters of adjustable process variable drifts, maintenance costs, and quality loss (estimated based on the cost of scraped parts) are calibrated based on data collected from real stamping processes. In this study, the production time is measured in terms of the number of work-pieces produced. The values in Table II are corresponding to the coded values used in statistical experimental design. The corresponding real values of these parameters are listed in Table III.

Based on the model parameters given above, and following the procedures in Sections II-B & II-C, $Q(\mathbf{s})$ & $E[Q(\mathbf{s}(t))]$ for the stamping process are obtained as

$$\begin{aligned} Q(\mathbf{s}) &= 1.83 + 2.19 \times 10^{-11} s_1^2 \\ &\quad + s_1(2.53 \times 10^{-6} + 5.06 \times 10^{-12} s_2) \\ &\quad + 2.92 \times 10^{-7} s_2 + 3.04 \times 10^{-13} s_2^2 \qquad (17) \\ E\left[Q\left(\mathbf{s}(t)\right)\right] &= 1.83 + 2.19 \times 10^{-11} E\left[s_1^2(t)\right] + E\left[s_1(t)\right] \\ &\quad \times (2.53 \times 10^{-6} + 5.06 \times 10^{-12} E\left[s_2(t)\right]) \\ &\quad + 2.92 \times 10^{-7} E\left[s_2(t)\right] \\ &\quad + 3.04 \times 10^{-13} E\left[s_2^2(t)\right] \qquad (18) \end{aligned}$$

Following the procedures in Section III, the optimization problem corresponding to the SBR policy, the MBR policy, and the AR policy are given in (19), (20), and (21), respectively.

$$\begin{aligned} b^* = \arg\min_b \Bigg(&\frac{e^{-11\times10^{-6}b}}{b} \\ &\times (-19\,184 + (237\,095 + 0.685b) \\ &\quad \times e^{3\times10^{-6}b} + (147\,859 + 0.0676) \\ &\quad \times e^{8\times10^{-6}b} \\ &\quad + (-359\,770 + 3.202b) e^{11\times10^{-6}b}) \Bigg) \\ \text{subject to } & b > 0 \qquad (19) \end{aligned}$$

$$\begin{aligned} (b^*, h^*) = \arg\min_{(b,h)} &\frac{e^{-11\times10^{-6}(b+h)}}{b} \\ \times \Big(&3.202\,19 b e^{11\times10^{-6}(b+h)} - 4.60 \times 10^{-7} \\ &\times (125\,000 + h)(333\,333 + h) + 4.60 \times 10^{-7} \\ &\times e^{11\times10^{-6}b}(125\,000 + h)(333\,333 + h) \\ &- 2.74 \times 10^{-6} e^{10^{-6}(11b+3h)}(160\,726 + h) \\ &\times (531\,696 + h) - 1.01 \times 10^{-7} e^{10^{-6}(11b+8h)} \\ &\times (355\,464 + h)(4.02 \times 10^6 + h) + e^{3\times10^{-6}(b+h)} \\ &\times \big(b(0.68 + 5.48 \times 10^{-6} h) \\ &\quad + 2.74 \times 10^{-6}(163\,703 + h)(528\,720 + h)\big) \\ &+ e^{8\times10^{-6}(b+h)} \\ &\times \big(b(0.068 + 2.03 \times 10^{-7} h) + 1.01 \times 10^{-7} \\ &\quad \times (363\,558 + h)(4.01 \times 10^6 + h)\big)\Big) \\ \text{subject to } & b > 0, \quad 0 \le h \le b \qquad (20) \end{aligned}$$

$$\begin{aligned} (a_1^*, a_2^*) = \arg\min_{(a_1,a_2)} &\frac{1}{(e^{8\times10^{-6}a_1} - 1)(e^{3\times10^{-6}a_2} - 1)} \\ \times \Big(&3.20 - 3.23 e^{8\times10^{-6}a_1} - 3.21 e^{3\times10^{-6}a_2} \\ &+ 3.24 e^{10^{-6}(8a_1+3a_2)} \\ &+ a_2(1.13 \times 10^{-6} - 1.13 \times 10^{-6} e^{8\times10^{-6}a_1}) \\ &+ a_2^2(3.04 \times 10^{-13} - 3.04 \times 10^{-13} e^{8\times10^{-6}a_1}) \\ &+ a_1(9.70 \times 10^{-6} + 5.06 \times 10^{-12} a_2 - 9.70 \\ &\quad \times 10^{-6} e^{3\times10^{-6}a_2}) \\ &+ a_1^2(2.19 \times 10^{-11} - 2.19 \times 10^{-11} e^{3\times10^{-6}a_2})\Big) \\ \text{subject to } & a_1 > 0, \quad a_2 > 0 \qquad (21) \end{aligned}$$

From (17), $Q(\mathbf{s})$ is strictly increasing. Further from Result 4, the optimization problem in (19) has a unique global optimum.

TABLE II
MAINTENANCE MODEL PARAMETERS FOR THE STAMPING PROCESS

σ_z	δ_1	δ_2	μ_1	μ_2	σ_{01}	σ_{02}
0.48	0.5	-0.5	$6.94\cdot 10^{-6}$	$4.17 \cdot 10^{-6}$	0.076	0.062
σ_1	σ_2	λ_1	λ_2	q	c_1^p	c_2^p
7.45×10^{-5}	4.47×10^{-5}	8×10^{-6}	3×10^{-6}	35	3000	3000

TABLE III
MAINTENANCE MODEL PARAMETERS IN TERMS OF REAL VALUES

σ_z (mm)	δ_1(in.)	δ_2 (in.)	μ_1(in.)	μ_2(in.)	σ_{01}(in.)	σ_{02}(in.)
0.03	83.6682	95.9488	5.97×10^{-8}	4.40×10^{-8}	6.54×10^{-4}	6.54×10^{-4}
σ_1 (in.)	σ_2 (in.)	λ_1	λ_2	q($/mm^2)	c_1^p ($)	c_2^p ($)
6.41×10^{-7}	4.72×10^{-7}	8×10^{-6}	3×10^{-6}	35	3000	3000

TABLE IV
OPTIMAL RESETTING POLICIES FOR THE STAMPING PROCESS

	SBR	MBR	AR
Optimal Policy	b^*=5.8×10^4	b^*=5.4×10^4, h=2.18×10^4	$a_1^* = 4.1\times10^4$, $a_2^* = 12.4\times10^4$
Minimum Long-run Average Cost ($/part)*	2.0244	2.0183	1.9915

*Because all the three resetting policies are independent of c_i^f, the long-run average costs in this table do not include the common cost component $\sum_{i=1}^{n} \lambda_i c_i^f$.

If it has a local optimum, then the local optimum must be a global optimum. From Corollary 3, a finite optimal solution for (20) also exists. The AR problem in (21) can be solved by exhaustive search because there are only two significant adjustable process variables in this system. The optimal solutions for all three policies are listed in Table IV. Because the AR policy is usually used as a benchmark to evaluate the cost-efficiency of the SBR & MBR policies, we focus on the selection of policies between SBR & MBR. As the long-run average costs of both SBR & MBR are close to that of the AR, their cost efficiency is both satisfactory. Because SBR is easier to implement, an SBR with a preventive resetting interval of 5.8×10^4 operations is preferred.

VI. SUMMARY, AND DISCUSSIONS

A QOM methodology is proposed in this paper for discrete-part manufacturing processes. The major challenge of the QOM research lies in the fact that multiple process variables in a manufacturing process generally have significant interactive impacts on the product quality loss. These interactive effects among adjustable process variables call for a multi-component maintenance model to jointly study optimal maintenance policies for all adjustable process variables. The model proposed in this paper focuses on easy-to-implement maintenance policies with consideration of both component catastrophic failures, and joint effects of component degradations on product quality loss. The response model intensively used in the literature of statistical experimental design is adopted in this paper to capture the general relationship between process variables and product quality. Three typical multi-component maintenance policies, i.e., simple block replacement, modified block replacement, and age replacement, are adapted for the QOM methodology. The age replacement strategy may be difficult to implement in practice because it requires different maintenance cycles for all components. However, it provides a benchmark to evaluate the cost-efficiency of other practical maintenance strategies. When the number of adjustable process variables is large, the computation load of solving the age resetting policy can be very high. However, in many manufacturing systems, not all adjustable process variables have significant impacts on product quality. Only the maintenance of tooling components relating to significant adjustable process variables need to be studied with consideration of product quality. Screening techniques in statistical experimental design can be used to efficiently select significant variables, and eliminate insignificant ones. When the number of significant adjustable process variables is small (say, ≤ 5), the nonlinear optimization problem for the age resetting policy can be solved by exhaustive search.

In this paper, the long-run average costs are derived for all three maintenance policies, and the optimal policy can be obtained by solving a nonlinear optimization problem. Properties of the optimal solutions for the SBR & MBR policies are investigated to facilitate optimization procedures. To calculate the best practical maintenance strategy, numerous process parameters need to be input into the formula developed in this paper. Design engineers who are familiar with process design & planning information should be in charge of calculating & selecting the best maintenance strategy. Once the desired maintenance strategy is obtained, SBR or the MBR policy will be easy to implement during the production phase because both policies use a joint, fixed maintenance schedule for different tooling components.

APPENDIX I
PROOF OF LEMMA 3

Proof: It is defined that $\Phi(b) = E[\text{cost during one preventive resetting interval}]/b$.

On each resetting interval, the total preventive resetting cost is always $\sum_{i=1}^{n} c_i^p$. The s-expected total failure resetting cost is $\sum_{i=1}^{n} c_i^f E[N_i(b)]$, where $N_i(t)$ is the number of catastrophic failures of component i during time interval (0, t). Because each component has a constant hazard rate λ_i, $N_i(t)$ follows a Poisson distribution with mean $\lambda_i t$, for $t < b$. So, the s-expected total failure resetting cost is $\sum_{i=1}^{n} c_i^f \lambda_i t$. The s-expected quality loss at time t is $E[Q(\mathbf{s}(t))]$, and the cumulative quality loss during the whole resetting interval is $\int_0^b E[Q(\mathbf{s}(t))]dt$. Summing up the preventive resetting cost, failure resetting cost, and the cumulative quality loss, we have $E[\text{cost during one preventive resetting interval}] = \sum_{i=1}^{n} c_i^p + b\sum_{i=1}^{n} c_i^f \lambda_i + \int_0^b E[Q(\mathbf{s}(t))]dt$, and

$$\Phi(b) = \frac{\sum\limits_{i=1}^{n} c_i^p + b\sum\limits_{i=1}^{n} c_i^f \lambda_i + \int_0^b E\left[Q\left(\mathbf{s}(t)\right)\right]dt}{b} = \sum_{i=1}^{n} c_i^f \lambda_i + \frac{c^p + \int_0^b E\left[Q\left(\mathbf{s}(t)\right)\right]dt}{b}. \qquad \square$$

APPENDIX II
PROOF OF LEMMA 7

If Lemma 7 is valid for the MBR, then by setting $h = 0$, it will be also valid for the SBR. Therefore, we only need to prove Lemma 7 for the MBR.

For $E[s_i(t)]$, from Lemma 4,

$$\frac{dE\left[s_i(t)\right]}{dt} = \frac{d}{dt}\left(\frac{1}{\lambda_i} - \left(\frac{1}{\lambda_i} + h\right)e^{-\lambda_i(h+t)}\right) = (1 + h\lambda_i)e^{-\lambda_i(h+t)} > 0$$

$$\frac{dE\left[s_i(t)\right]}{dh} = \frac{d}{dh}\left(\frac{1}{\lambda_i} - \left(\frac{1}{\lambda_i} + h\right)e^{-\lambda_i(h+t)}\right) = h\lambda_i e^{-\lambda_i(h+t)} \geq 0$$

Therefore, $E[s_i(t)]$ is an increasing function of both t & h.

For $Var[s_i(t)]$, from Lemma 4,

$$Var\left[s_i(t)\right] = \frac{1 - \left(h^2\lambda_i^2 + 2(h\lambda_i + 1)\lambda_i t\right)e^{-(h+t)\lambda_i} - (1 + h\lambda_i)^2 e^{-2(h+t)\lambda_i}}{\lambda_i^2}.$$

The equation above can also be considered as a function of $\lambda_i h$ & $\lambda_i t$. Therefore, without loss of generality, we can set $\lambda_i = 1$.

$$\frac{d}{dt}\left(1 - \left(h^2 + 2(h+1)t\right)e^{-(h+t)} - (1+h)^2 e^{-2(h+t)}\right) = e^{-2(h+t)}\left(2(h+1)^2 + e^{h+t}\left(h^2 + 2(t-1)(h+1)\right)\right) \qquad (22)$$

First, we show that $g_1(h) \equiv 2(1+h)^2 + e^{2h}(3h^2 - 2) \geq 0$. This can be seen by repeatedly calculating the derivatives of g as

$$g_1(0) = 0,\ \frac{dg_1}{dh}(0) = 0,\ \frac{d^2g_1}{dh^2}(0) > 0,\quad \frac{d^3g_1}{dh^3}(h) = 4e^{2h}(5 + 18h + 6h^2) > 0.$$

Also, we have

$$\frac{d}{dt}\left(2(h+1)^2 + e^{h+t}\left(h^2 + 2(t-1)(h+1)\right)\right) = e^{h+t}(h^2 + 2t + 2ht) \geq 0, \quad \forall h. \qquad (23)$$

When $t = h$,

$$2(h+1)^2 + e^{h+t}(h^2 + 2(t-1)(h+1)) = g_1(h) \geq 0. \qquad (24)$$

From (23) & (24),

$$2(h+1)^2 + e^{h+t}\left(h^2 + 2(t-1)(h+1)\right) \geq 0, \quad \forall t \geq h.$$

The above equation is equal to zero only if $t = h$. This follows from (22) that $Var[s_i(t)]$ is strictly increasing on t when $t \geq h$.

To show $Var[s_i(t)]$ is an increasing function of h, we have

$$\frac{d}{dh}\left(1 - \left(h^2 + 2(h+1)t\right)e^{-(h+t)} - (1+h)^2 e^{-2(h+t)}\right) = e^{-2(h+t)}h\left(2(h+1) + e^{h+t}(-2 + h + 2t)\right) \qquad (25)$$

First, we show that $g_2(h) \equiv 2 + 2h + e^h(h - 2) \geq 0$. This can be seen by repeatedly calculating its derivatives as

$$g_2(0) = 0, \quad \frac{d}{dh}g_2(0) > 0, \quad \frac{d^2}{dh^2}g_2(h) = e^h h \geq 0.$$

Also, we have

$$\frac{d}{dt}\left(2(h+1) + e^{h+t}(-2 + h + 2t)\right) = e^{h+t}(h + 2t) \geq 0, \ \forall h. \qquad (26)$$

When $t = 0$,

$$2(h+1) + e^{h+t}(-2 + h + 2t) = g_2(h) \geq 0, \qquad \forall h. \qquad (27)$$

From (26) & (27),

$$2(h+1) + e^{h+t}(-2 + h + 2t) \geq 0, \qquad \forall t, \quad \forall h.$$

Further, from (25), $Var[s_i(t)]$ is an increasing function of h.

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