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Decision field theory extensions for behavior modeling in dynamic environment using Bayesian belief network

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Abstract

Decision field theory (DFT), widely known in the field of mathematical psychology, provides a mathematical model for the evolution of the preferences among options of a human decision-maker. The evolution is based on the subjective evaluation for the options and his/her attention on an attribute (interest). In this paper, we extend DFT to cope with the dynamically changing environment. The proposed extended DFT (EDFT) updates the subjective evaluation for the options and the attention on the attribute, where Bayesian belief network (BBN) is employed to infer these updates under the dynamic environment. Four important theorems are derived regarding the extension, which enhance the usability of EDFT by providing the minimum time steps required to obtain the stabilized results before running the simulation (under certain assumptions). A human-in-the-loop experiment is conducted for the virtual stock market to illustrate and validate the proposed EDFT. The preliminary result is quite promising.

Keywords: Decision field theory; Human decision-making; Bayesian belief network; Preference

1. Introduction

Human decision-making process is extremely complex and ambiguous. A decision is made based on the economical aspect as well as the psychological aspect (e.g. irresolution, vacillation, inconsistency, lengthy deliberation, and distress) [9,21]. Extensive research has been conducted on understanding and modeling the human decision-making process, where it can be classified into three major categories including (1) economical decision-making [13,12,19,6], (2) psychological decision-making [5,15,1], and (3) synthetic engineering-based decision-making [11,14,16,8,7,10,20,23,18]. First, the economical approaches are well established, mostly based on the assumption that the decision-maker is rational. However, the economical approaches have limitation on its incapability to represent human cognitive natures (e.g. pressure, fatigue, and memory). To overcome this limitation, several psychology-based concepts have been proposed (utility, or subjec-

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tive value, and subjective probability) [4]. However, most of the psychology-based research mainly focuses on the human behaviors under the simple laboratory environment. These models had been criticized about their serious imperfections that people are seldom confined to the static laboratory decision problems [18]. Finally, the synthetic engineering-based approaches, which complement the economical and psychological approaches, employ a number of engineering methodologies and technologies to help reverse-engineer (understand and extract features from) human behaviors in complex and realistic environments. The human decision-making models in this category are the collection of the proper engineering techniques employed for each sub-module. However, validation of such comprehensive models (considering interactions between the sub-modules) with the real human decisions is difficult. In summary, all three approaches have pros and cons.

In order to develop a high-fidelity and comprehensive model of a human decision-making, all three approaches mentioned above need to be effectively integrated and complemented. For example, the economical and psychological approaches (bottom-up approaches) need to be enhanced and extended to cope with human behaviors in a highly complex environment. In addition, models (major structures) based on the engineering-based approaches (top-down approaches) need to be validated. In this paper, we tackle the first solution approach, where we intend to enhance decision field theory (DFT) (widely known theory in the field of mathematical psychology) [2,17,1] to cope with the dynamically changing environment.

DFT is a cognitive approach to human decision-making based on psychological rather than economical principles [1]. It provides a mathematical framework leading to understanding the cognitive mechanism of the human deliberation process in making decision under uncertainty [2]. DFT is distinguished from the previous mathematical approaches in that it is probabilistic and dynamic [22]. "Dynamic" here denotes that DFT considers "time" as a factor affecting the decision. In contrast, "dynamic" in this paper means that multiple and interdependent decisions are made in an autonomously changing environment [6]. DFT has been successfully applied across a broad range of cognitive tasks including sensory detection, perceptual discrimination, memory recognition, conceptual categorization, and preferential choice [1]. The original DFT is briefly described in the following section.

DFT describes the dynamic evolution of preferences among options during the deliberation time using the linear system formulation (see Eq. (1)).

$$P(t+h) = SP(t) + CMW(t+h) \tag{1}$$

In Eq. (1), $P(t)^T = [P_1(t), P_2(t), P_3(t)]$ represents the preference state where $P_i(t)$ represents the strength of preference corresponding to option i at time $t < T_D$ and T_D is the time that the final decision is made. The preference state is updated at every time step h. Each element is explained below.

- The stability matrix S provides the effect of the preference at the previous state (the memory effect) and the effect of the interactions among the options. In detail, the diagonal elements of S are the memory for the previous state preferences and off-diagonal elements are the inhibitory interactions among competing options (exemplary S will be shown in Section 3). Matrix S is assumed to be symmetric and the diagonal elements are assumed to have the same value. These assumptions ensure that each option has the same amount of memory and interaction effects. Furthermore, for the stability of this linear system, the eigenvalues λ_i of S are assumed to be less than one in magnitude ($|\lambda_i| < 1$).
- The value matrix M ($m \times n$ matrix, where m is the number of options, and n is the number of attributes) represents the subjective evaluations (perceptions) of a decision-maker for each option on each attribute. For example, product brochures or magazines provide consumers with objective facts. Given this objective information, readers obtain their own subjective evaluations, which constitute the M matrix. If the evaluation value changes according to the nature, the matrix M is constituted with multiple states.
- The weight vector $W(t)$ ($n \times 1$ vector, where n is the number of attributes) allocates the weights of attention corresponding to each column (attribute) of M. In the case that M is constituted with multiple states, each weight $w_j(t)$ corresponds to the joint effect of the importance of an attribute and the probability of a state. An important assumption of DFT is that the weight vector $W(t)$ changes over time according to a stationary stochastic process. This assumption allows us to derive four important theories regarding the expected preference values (see Section 4).

- The matrix C is the contrast matrix comparing the weighted evaluations of each option, $MW(t)$. If each option is evaluated independently, then C will be I (identity matrix). In this case, the preference of each option may increase simultaneously (see Eq. (1)). Alternately, the elements of the matrix C may be defined as $c_{ii} = 1$ and $c_{ij} = -1/(n-1)$ for $i \neq j$ where n is the number of options. For example, the contrast matrix C for the case with two options is shown below:

$$C = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

In this case (which is used in this paper), preference increase of one option lowers the preference of alternative options, and the sum of the elements of $CMW(t)$ ($m \times 1$ vector, where m is the number of options) is always zero.

As discussed above, the only component of DFT that is dynamically changing is the weight vector $W(t)$ (via random sampling), which we believe is not enough to represent the preferences in the dynamically changing environment (see Section 3 for more details). Therefore, the goal of this research is to propose an extension to DFT to represent the preferences in a dynamically changing environment. The rest of this paper is organized as follow. In Section 2, we discuss Bayesian belief network-based DFT extension for the dynamically changing environment. In Section 3, we demonstrate the significance of the proposed extensions using conceptual simulation (stock investment), by comparing results between DFT and EDFT. Section 4 then discusses theorems regarding the extension, which enhance the usability of EDFT by providing the stable results in the minimum amount of time. Section 5 then describes a human-in-the-loop experiment for virtual stock investment, which is used to compare and validate DFT and EDFT against the real human decisions. Finally, conclusions are presented in Section 6.

2. Bayesian belief network-based decision field theory extension

This section discusses two extensions to the original DFT that we propose based on the assumptions of the human behavior to cope with the dynamically changing environment. First, we assume that the subjective evaluations about each option (the value matrix M) may change during the decision deliberation. Second, the stochastic process of the attention weight may change according to the dynamically changing environment. Thus, the weight vector $W(t)$ may involve different stochastic processes.

2.1. DFT extension for dynamic changes of evaluation on options

In Eq. (1), the matrix M – the hypothetical subjective values of each option – is assumed to remain same or switch between predefined sets of values with some probability over the deliberation time. However, the values of matrix M may change during the decision deliberation more dynamically depending on the environment. In this case, the value matrix M is also dynamic over time. Thus, we propose the following extended model

$$P(t+h) = SP(t) + CM(t+h)W(t+h) \tag{2}$$

For the illustration purposes, the following exemplary decisions in stock investment will be used throughout this paper. In the exemplary stock investment, it is assumed that we (decision-makers) consider only two attributes – 'Investment safety' and 'Return'. It is noted that they are decision-maker's perceptions, not the objective values. As explained in Section 1, a decision-maker evaluates the options based on the given information and composes the value matrix M. Table 1 shows the hypothetical subjective values of each choice of stocks

Table 1
Hypothetical subjective values of options depending on time

Options	Time t		Time $t+h$	
	Investment safety	Return	Investment safety	Return
A	$S_A(t)$	$R_A(t)$	$S_A(t+h)$	$R_A(t+h)$
B	$S_B(t)$	$R_B(t)$	$S_B(t+h)$	$R_B(t+h)$

(options) in the stock market example. Since the values of these attributes are subjective, they may change easily over time depending on the environmental condition. For instance, Table 1 depicts that initial evaluation for the option A on the 'Investment safety' attribute at time t, $S_A(t)$, is changed to $S_A(t+h)$ at time $t+h$.

Thus, the value matrix M has the dynamic representation as shown at Eq. (3) (see Eq. (1) for comparison) and it changes during the deliberation time

$$M(t) = \begin{bmatrix} S_A(t) & R_A(t) \\ S_B(t) & R_B(t) \end{bmatrix} \tag{3}$$

2.2. DFT extension for dynamic changes of attention weights

As the environment changes dynamically, the attention weight of a decision-maker may also change. This is reflected to the change of the weight vector $W(t)$ over time in DFT. Regarding $W(t)$, Roe et al. [17] assumed that the weights are identically and independently distributed (iid) over time, where the dynamic change of the environment is not considered in the weight vector. Similarly, Diederich [3] used a Markov process to represent the switch between sub-processes which are individually iid. In the Markov process, the moment at which the sub-processes switch over can be considered as the moment when the environment related to the decision making changes. However, the changes of the sub-process in the Markov process are based on the probability, not the current environment. For this reason, the Markov process is insufficient to cope with the dynamic environment. Therefore, it is necessary to employ a richer technique accounting for the dynamic environment.

2.3. Bayesian belief network-based extension

In this research, Bayesian belief network (BBN) is employed to incorporate the extensions discussed in Sections 2.1 and 2.2. In other words, BBN enable EDFT to model (1) the change of evaluation on the options and (2) the change of human attention along with the dynamically changing environments (see Fig. 1). Based on the given information and the previous history, BBN infers the distribution of value matrix $M(t)$ and weight vector $W(t)$. More details about BBN are discussed below.

BBN (see Fig. 2) is a cause and effect network that captures the probabilistic relationship, as well as historical information. BBN contains prior and conditional probabilities that can be used to infer the posterior probability through the Baye's theorem. A major advantage of BBN is that it can provide valuable analysis under uncertainty. Even when the given information is imperfect, BBN still can give a very convincing answer based on the historical data. This flexibility of BBN enables EDFT to content with the uncertainty of the real world.

Fig. 2 depicts an instance of BBN for the stock market situation. The directed links represent the cause and effect relationship. For example, a decision-maker's 'Investment history' (the return on investment in the past) affects his/her attention on 'Investment safety' in Fig. 2. In the real stock market, the attention on the 'Investment safety' attribute of a decision-maker may be affected by numerous factors. In this paper, however, we

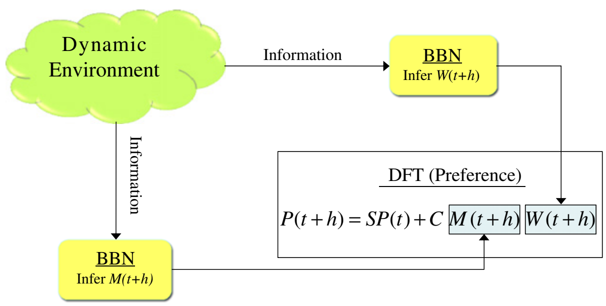

Fig. 1. BBN-based EDFT for dynamically changing environment.

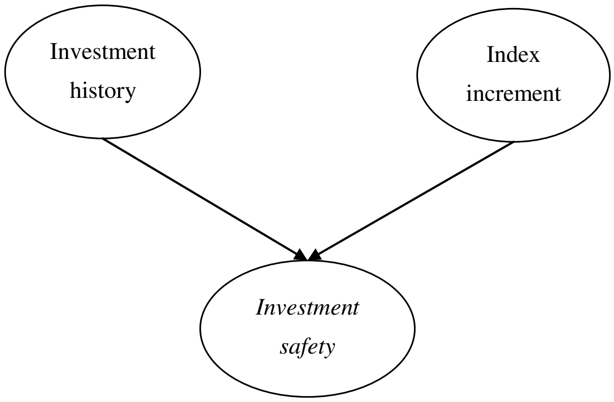

Fig. 2. Bayesian belief network for stock investment.

consider a limited set of affecting factors, including the past investment history (Investment history) and the increment of stock index (Index increment). It is noted that more factors can be considered in the similar manner. In Fig. 2, each node represents the event that can occur. The distribution of node can be either discrete or continuous. In this paper, we used the discrete distributed node for the sake of implementation. Thus each node has countable states. If there is a link from node A to B, it is said that B is a child of A and A is a parent of B. In Fig. 2, 'Investment history' and 'Index increment' are the parents and 'Investment safety' is the child. In order to build a complete BBN, the probability distribution of each parent and the conditional probability between the states of each parent and child are necessary. These prior probabilities in BBN are attained (trained) through the human-in-the-loop experiment (see Section 5 for more details) so that the distribution change of the weight vector ($W(t)$) and the change of $M(t)$ matrix can mimic the real human's behavior.

3. Significance of the proposed extensions

This section demonstrates the significance of the proposed extensions using conceptual simulation (stock investment) in Matlab, by comparing results between DFT and EDFT. In the considered simulation, the time unit is 'day' and the time step h is defined as one day. And, the deliberation time is fixed to 100 days, and the initial preference $P(0)$ is set to 0 vector. Fig. 3 shows two options of the stock market example, where each option is characterized with 'Investment safety' and 'Return'. As mentioned before, it is noted that 'Investment safety' and 'Return' are the perception of each individual, not the true, objective values. In Fig. 3, X-axis

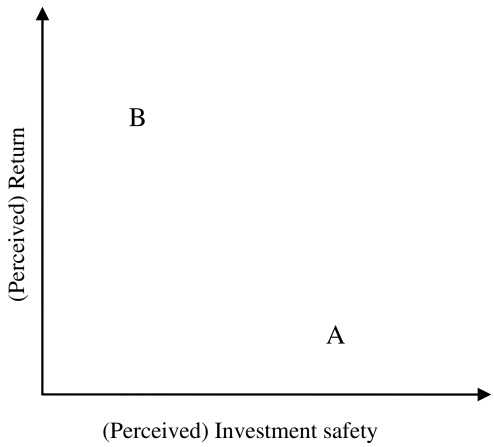

Fig. 3. A graphical depiction of two options in the stock market example.

represents the investment safety, and high value of 'Investment safety' means small investment risk. Similarly, Y-axis represents the return from the investment. The ranges of both attribute value are set from 0 to 5.

In this example, options A and B are not close each other, and therefore they do not have large interaction between them. Thus, the values of off-diagonal elements in the S matrix are relatively small. The memory effect is set to decay slowly by setting a high value to the diagonal elements. Considering these, we can define S as following:

$$S = \begin{pmatrix} 0.9 & -0.01 \\ -0.01 & 0.9 \end{pmatrix}$$

3.1. Effect of change in value matrix $(M(t))$

In this section, we investigate the impact of change of $M(t)$ on the preference states in the DFT. Table 2 depicts that the values of $M(t)$ change at day 51, and Eqs. (4) and (5) depict the corresponding $M(t)$ and the preference model, respectively. For the weight vector $W(t)$, we used a simple Bernoulli process. It is assumed that the weight changes in an all-or-none manner from one attribute to another with some probabilities. The considered probabilities in our simulation are $\Pr(W_{\text{Investment safety}} = 1) = 0.45$, $\Pr(W_{\text{Investment safety}} = 1) = 0.43$. And, with probability 0.12 none of the two attributes attains the weight.

$$M(t) = \begin{cases} \begin{pmatrix} 3.5 & 1.3 \\ 1.3 & 3.5 \end{pmatrix} & \text{if } t \leqslant 50 \\ \begin{pmatrix} 3.4 & 1.3 \\ 1.3 & 3.5 \end{pmatrix} & \text{if } 50 \leqslant t \end{cases} \tag{4}$$

$$P(t+1) = \begin{pmatrix} p_1(t+1) \\ p_2(t+1) \end{pmatrix} = \begin{cases} \begin{pmatrix} 0.9 & -0.01 \\ -0.01 & 0.9 \end{pmatrix}\begin{pmatrix} p_1(t) \\ p_2(t) \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 3.5 & 1.3 \\ 1.3 & 3.5 \end{pmatrix}\begin{pmatrix} w_1(t+h) \\ w_2(t+h) \end{pmatrix} & \text{if } t \leqslant 50 \\ \begin{pmatrix} 0.9 & -0.01 \\ -0.01 & 0.9 \end{pmatrix}\begin{pmatrix} p_1(t) \\ p_2(t) \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 3.4 & 1.3 \\ 1.3 & 3.5 \end{pmatrix}\begin{pmatrix} w_1(t+h) \\ w_2(t+h) \end{pmatrix} & \text{if } 50 < t \end{cases} \tag{5}$$

The preference model shown in Eq. (5) has been simulated until 100th day for 2000 independent replications. At each time t, the option which has a higher preference is considered to be chosen. Then, we can calculate the choice probability of each option by counting the frequency of being selected (see Fig. 4). In Fig. 4, the choice probabilities over time are compared between the case with dynamic $M(t)$ (see Eq. (5)) and the static $M(t)$ ($M(t)$ does not change at day 51). In the figure, the dotted lines, indicated with "+" and "o" symbols, represent the choice probabilities for options A and B for the static $M(t)$, respectively. Similarly, the solid lines indicated with "×" and "□" symbols represent the choice probabilities for options A and B for the dynamic $M(t)$, respectively. As shown in Fig. 4, the outcomes at day 100 (when a decision time is made) are different between two cases. While the probability for option A is higher than option B for the static case, the result is opposite for the dynamic case. Therefore, it is found that even a slight change of value of M (m_{11} changes 3.5–3.4) can make different results, and the choice probability is very sensitive to the value of matrix M. This supports the motivation and importance of the proposed research on considering the dynamic case.

Table 2
The value matrix $M(t)$ used in the simulation

Options	Day 0		Day 51	
	Investment safety	Return	Investment safety	Return
A	3.5	1.3	3.4	1.3
B	1.3	3.5	1.3	3.5

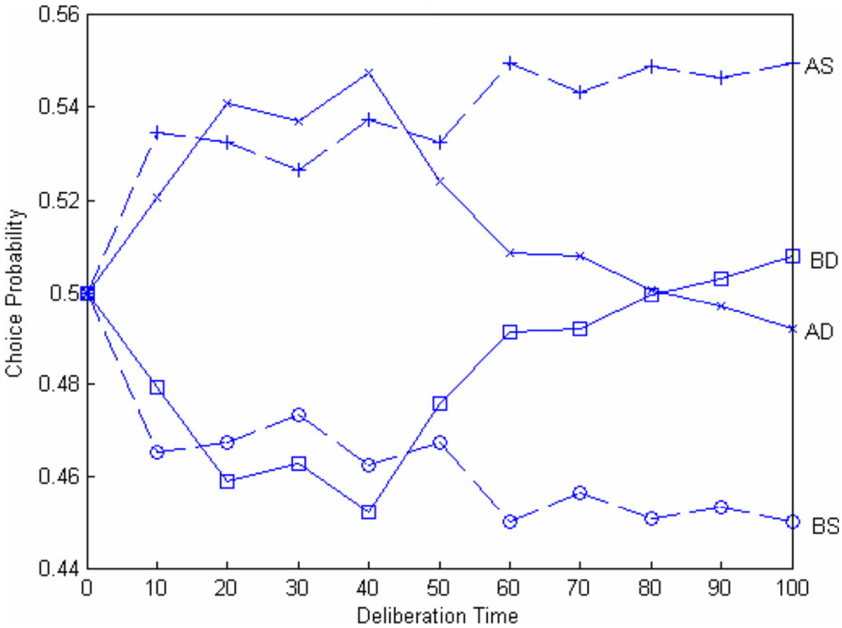

Fig. 4. Comparison of choice probabilities between static $M(t)$ and dynamic $M(t)$.

3.2. Effect of change in weight vector ($W(t)$)

In the simulation considered in this section, the $M(t)$ matrix is static, but $W(t)$ probabilities change (see Eq. (6)) during the decision deliberation. Fig. 5 depicts the effect of the $W(t)$ change on the choice probability. The same notations used in Fig. 4 are applicable to Fig. 5. As shown in Fig. 5, the effect of the change of $W(t)$ is even bigger then the effect of the change of $M(t)$ (see Fig. 4).

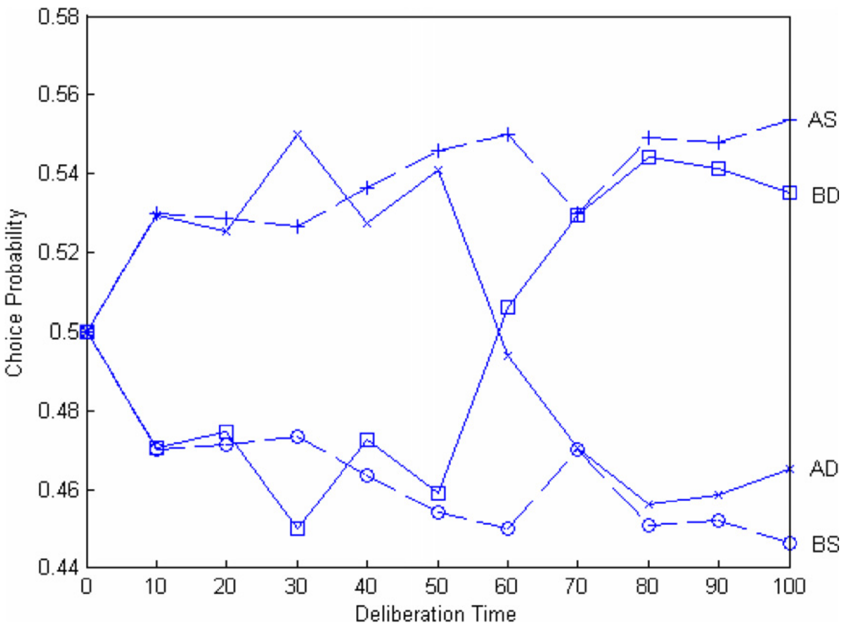

Fig. 5. Comparison of choice probabilities between static $W(t)$ and dynamic $W(t)$.

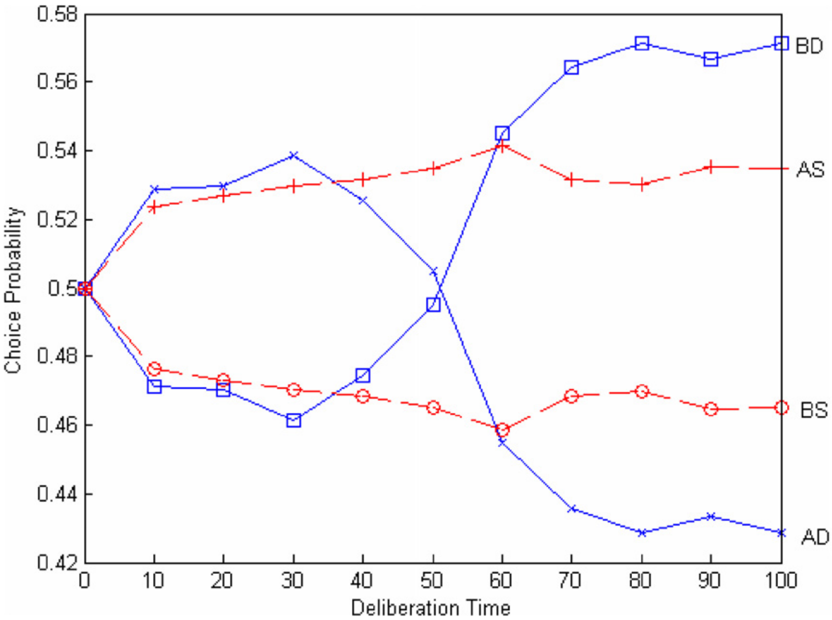

Fig. 6. Comparison of choice probabilities between static $M(t)$ and $W(t)$ and dynamic $M(t)$ and $W(t)$.

$$\begin{aligned} &\Pr(W_{\text{Investment safety}} = 1) = \begin{cases} 0.45 & \text{if } t \leqslant 50 \\ 0.43 & \text{if } 50 < t \end{cases} \\ &\Pr(W_{\text{Return}} = 1) = \begin{cases} 0.43 & \text{if } t \leqslant 50 \\ 0.45 & \text{if } 50 < t \end{cases} \end{aligned} \tag{6}$$

3.3. Combined effect of changes in $M(t)$ and $W(t)$

In the simulation considered in this section, the $M(t)$ matrix is dynamic (see Eq. (5)) and $W(t)$ probabilities change (see Eq. (6)) during the decision deliberation. Fig. 6 depicts the choice probabilities for the considered simulation, where the combined effect has been boosted up to invert and increase the choice probability. Again, significance of changes in $M(t)$ and $W(t)$ has been shown in Sections 3.1, 3.2, and 3.3, and these results support the motivation and importance of the proposed research on considering the dynamic case.

4. Four theorems regarding expected preference values

The expected preference values (over the multiple replications) for each option at a given time gives us an idea which option will be chosen more frequently before actual deploying of DFT if the difference between the expected values for each option is large. In this section, we introduce four important theorems about the expected preference values of the two options decision-making problem. These theorems tremendously enhance the usability of DFT because it provides us with the minimum amount of time steps needed for the preference values to be stabilized before we actually run the simulation (evolve the DFT).

Theorem 1. *In the two options decision-making problem of the original DFT, the expected value of preference is*

$$E(P(nh)) = \frac{1-D^n}{1-D} E(v_1(h)) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

where $D = s_{11} - s_{12}$ and $E(v_1(h)) = E(w_1(h))(m_{11} - m_{21}) + E(w_2(h))(m_{12} - m_{22})$.

Furthermore

$$E(P(nh)) = \frac{1}{1-D} E(v_1(h)) \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{as } n \to \infty$$

Proof. Suppose the valence $V(t) = CMW(t)$. Then $P(t+h) = SP(t) + V(t+h)$. By assumption of DFT, we get $S = \begin{pmatrix} s_1 & s_2 \\ s_2 & s_1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ in the two options case. In most applications, the initial preference state $P(0) = 0$ [1]. Thus we get $p_1(h) = s_1 p_1(0) + s_2 p_2(0) + v_1(h) = v_1(h)$. So, the expected value of $p_1(h)$ is $E(p_1(h)) = E(v_1(h))$. From the definition of $V(t)$, $E(v_1(h)) = E(w_1(h)(m_{11} - m_{21}) + w_2(h)(m_{12} - m_{22})) = E(w_1(h))(m_{11} - m_{21}) + E(w_2(h))(m_{12} - m_{22})$. Since the weight vector $W(t)$ changes over time according to a stationary stochastic process (see Section 1),

$$E(v_1(h)) = E(v_1(ih)) = E(p_1(h)) \quad \text{for all } i > 1 \tag{7}$$

Now $E(p_1(2h)) = E(s_1 p_1(h) + s_2 p_2(h) + v_1(2h)) = (s_1 - s_2)E(p_1(h)) + E(v_1(2h))$.

Let $D = s_1 - s_2$ then from (7), $E(p_1(2h)) = (s_1 - s_2 + 1)E(p_1(h)) = (D+1)E(p_1(h))$. Similarly $E(p_1(3h)) = E(s_1 p_1(2h) + s_2 p_2(2h) + v_1(3h)) = (s_1 - s_2)E(p_1(2h)) + E(v_1(3h)) = DE(p_1(2h)) + E(p_1(h)) = D(D+1)E(p_1(h)) +$ $(2h)) + E(v_1(3h)) = DE(p_1(2h)) + E(p_1(h)) = D(D+1)E(p_1(h)) + E(p_1(h)) = (D^2 + D + 1)E(p_1(h))$.

By induction,

$$E(p_1((n+1)h)) = DE(p_1(nh)) + E(p_1(h)). \tag{8}$$

Thus $E(p_1(nh)) = \sum_{i=0}^{n-1} D^i E(p_1(h))$. Since $0 < D < 1$ from the assumption of S,

$$E(p_1(nh)) = \frac{1 - D^n}{1 - D} E(p_1(h)) = \frac{1}{1 - D} E(p_1(h)) \quad \text{as } n \to \infty. \tag{9}$$

From Eq. (7) and $p_1(nh) + p_2(nh) = 0$, the expected preference of DFT is $E(P(nh)) = \frac{1-D^n}{1-D} E(v_1(h)) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. □

Theorem 1 informs us the following fact. In the two choices problem, the sign of the value $E(v_1(h))$ only determines which option has bigger expected preference values. For example, if $E(v_1(h)) > 0$ then $E(p_1(t)) > E(p_2(t))$. And if $E(v_1(h)) < 0$ then $E(p_1(t)) < E(p_2(t))$. However, it does not necessarily mean that the option having a higher expected preference has a higher choice probability which is calculated by counting the number of cases involving a higher preference. The option which has a higher choice probability is determined by the median of the sampled preference. The option having a higher median will have a higher choice probability. We can also notice that the expected preference value converges as n increases. This property leads us to the following Theorem 2.

Theorem 2. *The difference between the expected preference value* $E(P(nh))$ *and its converging value becomes less than* $\varepsilon > 0$ *after n time step where* $n = \left\lceil \frac{\log k}{\log D} \right\rceil$ *and* $k = \frac{1-D}{|E(v_1(h))|} \varepsilon$.

Proof. Suppose n is large enough so that the difference is less than ε. Then from Eq. (9) we obtain $\left| \frac{1}{1-D} E(p_1(h)) - \frac{1-D^n}{1-D} E(p_1(h)) \right| \leqslant \varepsilon$. Thus $\frac{D^n}{1-D} |E(p_1(h))| \leqslant \varepsilon$. Finally we have $D^n \leqslant \frac{1-D}{|E(p_1(h))|} \varepsilon$. Let $k = \frac{1-D}{|E(v_1(h))|} \varepsilon$. By taking logarithm on both side we get $n \geqslant \frac{\log k}{\log D}$. □

From Theorems 1 and 2, we can see that after some time step n the expected preference values become steady. This also infers that the choice probability of each option will not change after some time step n. Thus we can attain a steady preference value after n steps of evolution. To illustrate the above discussions, we consider the stock example, where $S = \begin{pmatrix} 0.9 & -0.01 \\ -0.01 & 0.9 \end{pmatrix}$, $M = \begin{pmatrix} 3.5 & 1.3 \\ 1.3 & 3.5 \end{pmatrix}$, and $\Pr(W_{\text{Investment safety}} = 1) = 0.53$ and $\Pr(W_{\text{Return}} = 1) = 0.47$. From Theorem 1, $D = 0.9 - (-0.01) = 0.91$ and $E(v_1(h)) = 0.53(3.5 - 1.3) + 0.47(1.3 - 3.5) = 0.132$. Thus, the value of the expected preference of stock A converges to 1.4667. In Theorem 2, if we set $\varepsilon = 0.01$, then $k = 0.0068$ and $n = 53$. To validate these results, we executed the Matlab simulation for 10,000 replications, and the results are depicted in Fig. 7. As shown in Fig. 7, after time 53 the expected value reaches to ε neighbor of its convergence and the choice probabilities are stabilized. Therefore, the results from the theorems are valid.

Now let us consider EDFT. In EDFT the weight vector $W(t)$ and the value matrix $M(t)$ changes over time. Let n_i be the number of time steps after $(i-1)$th $V(t)$ changes and $V^i(t)$ be the ith changed valence. Then the following Theorem 3 can be derived from Theorem 1.

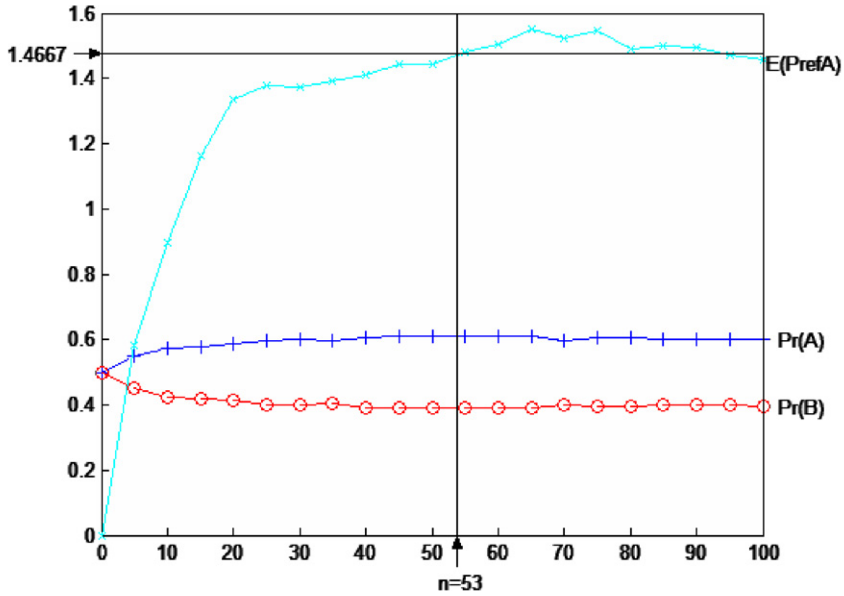

Fig. 7. Steady choice probability and time steps to the convergence of the expected preference values (simulation results).

Theorem 3. *Thus the expected preference of EDFT is*

$$E\left(P\left(\sum_{i=1}^{q} n_i h\right)\right) = \left\{\sum_{i=1}^{q-1}\left[D^{\sum_{j=i+1}^{q} n_j}\left(\frac{1-D^{n_i}}{1-D}\right)E(v_1^i(t))\right] + \left(\frac{1-D^{n_q}}{1-D}\right)E(v_1^q(t))\right\}\begin{pmatrix}1\\-1\end{pmatrix}$$

where q is the total number of changes of $V(t)$, $D = s_1 - s_2$, n_i is the number of time steps after $(i-1)$th change of $V(t)$, and $V^i(t)$ be the ith valence for $i > 1$.

Furthermore

$$E\left(P\left(\sum_{i=1}^{q} n_i h\right)\right) = \left(\frac{1}{1-D}\right)E(v_1^q(t))\begin{pmatrix}1\\-1\end{pmatrix} \quad as\ n_q \to \infty$$

Proof. From Theorem 1 we get $E(p_1(n_1 h)) = \frac{1-D^n}{1-D}E(v_1^1(t))$. However since $V(t)$ changes at $n_1 h$, from (8) we get $E(p_1((n_1+1)h)) = D\sum_{i=0}^{n_1-1} D^i E(v_1^1(t)) + E(v_1^2(t))$ and $E(p_1((n_1+2)h)) = D^2\sum_{i=0}^{n_1-1} D^i E(v_1^1(t)) + \sum_{i=0}^{1} D^i E(v_1^2(t))$.

Thus

$$E(p_1((n_1+n_2)h)) = D^{n_2}\sum_{i=0}^{n_1-1} D^i E(v_1^1(t)) + \sum_{i=0}^{n_2-1} D^i E(v_1^2(t)) = D^{n_2}\left(\frac{1-D^{n_1}}{1-D}\right)E(v_1^1(t)) + \left(\frac{1-D^{n_2}}{1-D}\right)E(v_1^2(t))$$

By induction

$$E(p_1((n_1+n_2+\cdots+n_q)h)) = \sum_{i=1}^{q-1}\left[D^{\sum_{j=i+1}^{q} n_j}\left(\frac{1-D^{n_i}}{1-D}\right)E(v_1^i(t))\right] + \left(\frac{1-D^{n_q}}{1-D}\right)E(v_1^q(t)).$$

Thus the expected preference of EDFT is

$$E\left(P\left(\sum_{i=1}^{q} n_i h\right)\right) = \left\{\sum_{i=1}^{q-1}\left[D^{\sum_{j=i+1}^{q} n_j}\left(\frac{1-D^{n_i}}{1-D}\right)E(v_1^i(t))\right] + \left(\frac{1-D^{n_q}}{1-D}\right)E(v_1^q(t))\right\}\begin{pmatrix}1\\-1\end{pmatrix}$$

where $D = s_1 - s_2$ and n_i is the number of time steps after $(i-1)$th $V(t)$ changes and $V^i(t)$ be the ith valence for $i > 1$. □

Theorem 3 induces the following Theorem 4.

Theorem 4. *If EDFT evolves enough to converge whenever the weight vector $W(t)$ or the value matrix $M(t)$ changes, then the difference between the expected preference value $E\left(P\left(\sum_{i=1}^{q} n_i h\right)\right)$ of EDFT and its converging value becomes less than $\varepsilon > 0$ after n_q time step since the last change of the weight vector $W(t)$ or the value matrix $M(t)$ where $n_q = \left\lceil \frac{\log k}{\log D} \right\rceil$ and $k = \frac{1-D}{|E(v_1^q(h)) - E(v_1^{q-1}(h))|}\varepsilon$.*

Proof. Suppose n is large enough so that the difference is less than ε. Then from Theorem 3 we have

$$\varepsilon \geqslant \left| \frac{1}{1-D} E(v_1^q(h)) - \left\{ \sum_{i=1}^{q-1} \left[D^{\sum_{j=i+1}^{q} n_j} \left(\frac{1-D^{n_i}}{1-D} \right) E(v_1^i(t)) \right] + \left(\frac{1-D^{n_q}}{1-D} \right) E(v_1^q(t)) \right\} \right|$$

$$= \left| \frac{D^{n_q}}{1-D} E(v_1^q(h)) - \sum_{i=1}^{q-1} \left[D^{\sum_{j=i+1}^{q} n_j} \left(\frac{1-D^{n_i}}{1-D} \right) E(v_1^i(t)) \right] \right|.$$

Since we assume that EDFT iterate enough to converge with each given $V(t)$, $D^{n_i} \to 0$ for $i < q$. Thus $\varepsilon \geqslant \frac{D^{n_q}}{1-D} |E(v_1^q(t)) - E(v_1^{q-1}(t))|$.

Finally we have $D^{n_q} \leqslant \frac{1-D}{|E(v_1^q(t)) - E(v_1^{q-1}(t))|}\varepsilon$. Let $k = \frac{1-D}{|E(v_1^q(t)) - E(v_1^{q-1}(t))|}\varepsilon$. By taking logarithm on both side we get $n_q \geqslant \frac{\log k}{\log D}$. □

Theorems 3 and 4 inform that after some time steps n_i the expected preference values are stabilized also in EDFT. The choice probabilities of each option remain same after some time steps n_i. Here, we consider the stock example used earlier in this section (see Fig. 7), where the value matrix $M(t)$ changes from $M(t) = \begin{pmatrix} 3.5 & 1.3 \\ 1.3 & 3.5 \end{pmatrix}$ to $M(t) = \begin{pmatrix} 3.4 & 1.3 \\ 1.3 & 3.5 \end{pmatrix}$ and the probabilities of weight vector changes from $\Pr(W_{\text{Investment safety}} = 1) = 0.53$ and

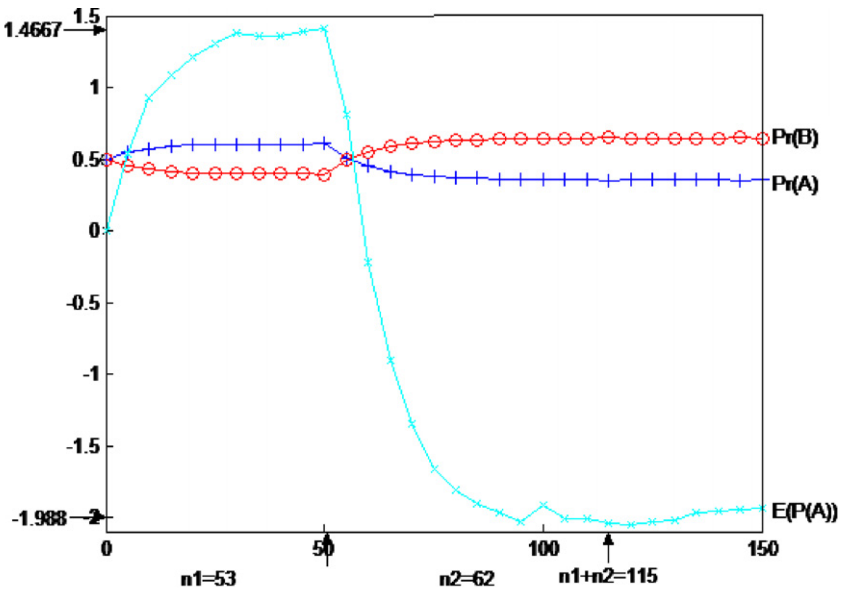

Fig. 8. Steady choice probability and time steps to the convergence of the expected preference values at dynamically changing environment (simulation results).

$\Pr(W_{\text{Return}} = 1) = 0.47$ to $\Pr(W_{\text{Investment safety}} = 1) = 0.47$ and $\Pr(W_{\text{Return}} = 1) = 0.53$. Then we can calculate $E(v_1^1(h)) = 0.132$ and $E(v_1^2(h)) = 0.47(3.4 - 1.3) + 0.53(1.3 - 3.5) = -0.179$. By Theorem 3 the value of the expected preference of stock A converges to -1.988. If we set $\varepsilon = 0.01$ in Theorem 4, then $k = 0.0029$ and $n_2 = 62$. To validate these results, we executed the Matlab simulation for 10,000 replications, and the results are depicted in Fig. 8. As shown in Fig. 8, at time 53 the expected preference value $(E(P(A)))$ reaches the ε neighbor of its first convergence (1.4667). Then, it continues to evolve with the changed $M(t)$ and $W(t)$ for next 62 time steps until time 115, when the expected preference value $(E(P(A)))$ reaches the ε neighbor of its second convergence (-1.988) and the choice probabilities are stabilized.

5. Human-in-the-loop experiment

In Section 2, we assumed that both the value matrix $M(t)$ and attention matrix $W(t)$ may change during the deliberation time against the dynamic environment. To test our assumptions, we designed and conducted a human-in-the-loop experiment. More specifically, the goal of the experiment is to find (1) how the evaluations on the each option $(M(t))$ are changed, and (2) how the attention weight of the decision-maker $(W(t))$ is changed. The experimental results are also used to characterize BBN (conditional probabilities between the nodes).

5.1. Human-in-the-loop experiment details

Software has been developed to allow a human-in-the-loop experiment involving a virtual stock market, where a daily stock index and current and historical price of each stock option are considered. The daily stock index and price are generated randomly from normal distributions. The stock index and price have been arranged so that they are positively correlated. This is the same example used in the previous sections. Fig. 9 is the screen capture of the virtual stock market program used in the experiment. Only two stock options (A and B) are considered in the experiment. Each experiment is continued until the subject makes 10 decisions. At each decision in the experiment, the subject is asked to choose a stock option within the decision deliberation

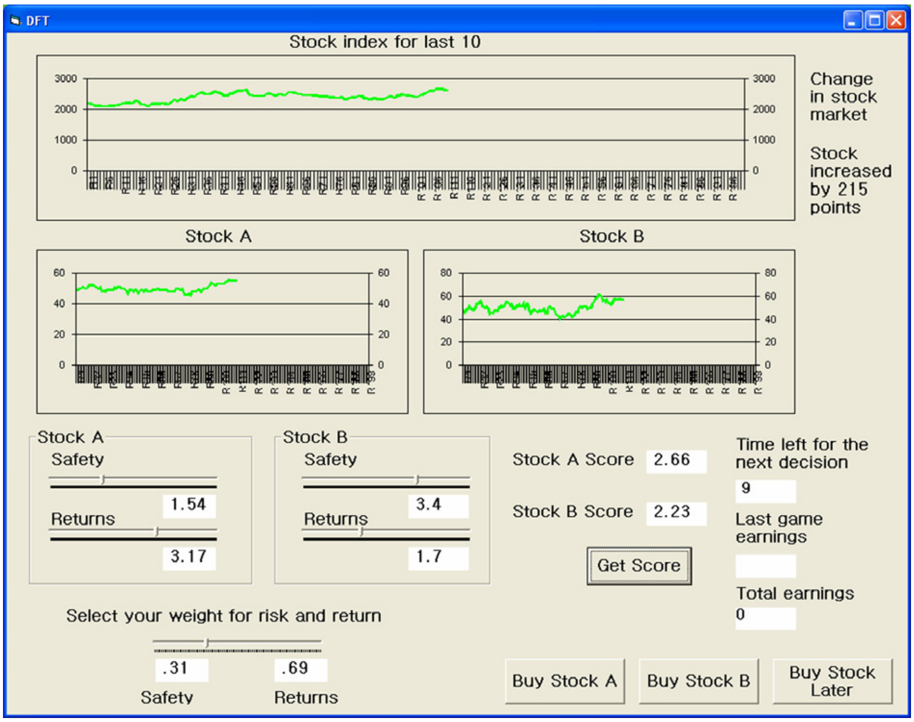

Fig. 9. Screen capture of the virtual stock trading software used in the experiment.

time (10 time units). In this work, we adopted the *fixed stopping time* rule which forces the decision-maker to make a decision within the given time. To this end, at the beginning of each time unit the subject can choose one from following three options: (1) 'buy stock A' now, (2) 'buy stock B' now or (3) 'buy later' (postpone the decision to the next time unit). At each time unit, the subject is also asked to express his/her perception on (1) the evaluation of each option on two attributes – 'Investment safety' and 'Return' ($M(t)$) and (2) the weights of attention on each attribute ($W(t)$). By decreasing the other weight value when one weight value is increased, the summation of attention weights on each attribute is set to be 1 all the time. The collected data is used to attain the conditional probabilities and the distributions in BBN (see Section 2.3). Based on the evaluation and the weight values received from the subject, the scores of each stock option are calculated using the multiplication of $M(t) \cdot W(t)$ and are shown to the subject. It is believed that showing these scores helps to draw the subject's attention into the experiment, and also for them to choose an option. If the subject chooses alternatives (1) or (2), the experiment continues with next decision with a different environment (stock market index value, prices of stocks A and B). If the subject chooses alternative (3), the experiment proceeds to the next time period and the market environment evolves. At the end of the given deliberation time (beginning of the 10th time unit), the subject is enforced to make a decision. In this case the decision-maker has only two alternatives – buy stock A or B (he/she cannot postpone the decision anymore). When the subject makes a decision (e.g. at the 5th time unit), the stock market continues to evolves till the 10th time unit, when the stock is sold and the monetary margin between the buy price (at the 5th time unit) and the sell price (at the 10th time unit) of the option is recorded as the reward for the subject. Each experiment is continued until the subject makes 10 decisions (100th time unit), during which the rewards are accumulated as 'Investment history'. We repeat this experiment for 10 times.

5.2. Experimental results and analyses

From the human-in-the-loop experiment (see Section 5.1), we observed that the value matrix $M(t)$ changed in the dynamically changing environment. For example, when the stock price has increased significantly, the evaluation (perception) of that stock (option) on the 'Return' attribute has increased. In the experiment, the stock price increased by around 17% at the moment when the evaluation value of 'Return' increased. Note that, on the average, the stock price changed within 0.1% of the price throughout the experiment. We also observed that the attention on the attributes 'Investment safety' and 'Return' ($W(t)$) were affected by the 'Index increment' which is the difference between index values of previous and current stock markets. Fig. 10 depicts an approximate linear relationship between the weight increment on 'Return' attribute and

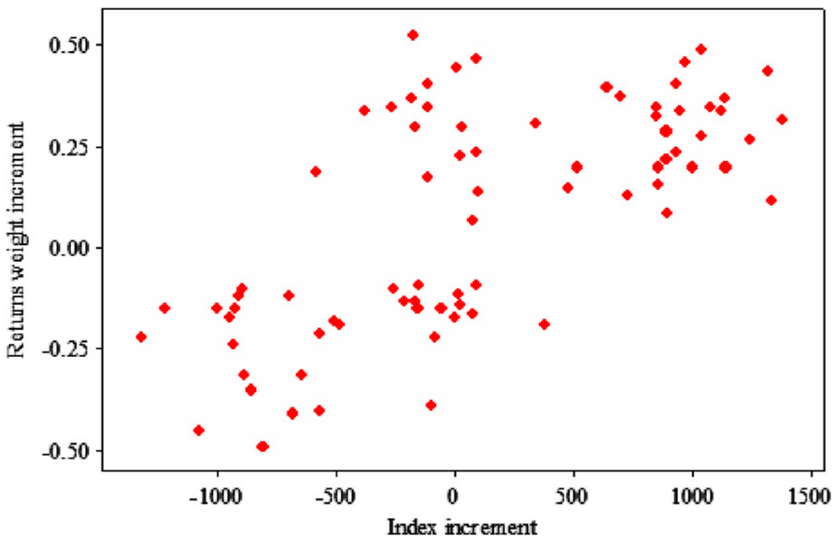

Fig. 10. Relationship between 'Return' weight increment and 'Index increment'.

'Index increment'. The correlation value between them is 0.68. Since the summation of 'Investment safety' and 'Return' is set to be 1, there is also approximate negative linear relationship between the weight increment on 'Investment safety' and 'Index increment'

5.3. Validation and comparison of EDFT with DFT and human decisions

As discussed in Section 2.3 and Fig. 2, the proposed EDFT model uses BBN to infer the changes of the value matrix $M(t)$ and the weight vector $W(t)$. This section discusses BBN which has been built from the human-in-the-loop experiment data (see Section 5.1). As discussed in Section 5.2, we observed the relationships among several factors, including 'Index increment', 'Investment history', 'Return', and 'Investment safety'. Based on this analysis, we have constructed a BBN representing the decision behavior of the subject who participated in the experiment (see Fig. 11). For each event (node) of BBN, we divided the range of collected data into three discrete states – high, medium, and low. Then, we calculated the conditional distribution of each descendant node given the state of the parent nodes. Table 3 depicts the conditional probability between nodes 'Index increment', 'Investment history' and 'Investment safety'. It is the conditional probability distribution of three discrete states in the 'Investment safety' node given the states of the parent nodes ('Index increment' and 'Investment history'). Other conditional probabilities are calculated in the similar manner. As mentioned before, the BBN shown in Fig. 11 was built based on the experiment in Section 5.1. Thus, it can be used to infer the value matrix $M(t)$ and the weight vector $W(t)$ only for the subject who had participated in the experiment. Different BBNs will need to be built and used to infer $M(t)$ and $W(t)$ for different individuals.

In order to validate the constructed BBN (which is used as part of the EDFT), we conducted one additional human-in-the-loop experiment involving 10 decisions (100 time units) after constructing the model. In this experiment, at each time unit the same environment (stock market index value, prices of stocks A and B) is presented to the human subject, DFT simulation model, and EDFT simulation model. And, all three of them are asked to choose a stock option (make a decision) within the decision deliberation time (10 time units). In the simulations of DFT and EDFT, the initial preference $P(0)$ is set to 0 and S is set to $S = \begin{pmatrix} 0.2 & -0.01 \\ -0.01 & 0.2 \end{pmatrix}$ since this S gives the most similar choice with the human subject. Also, based on Theorems 2 and 4 in Section 4, we used $\boldsymbol{\varepsilon} = 0.01$ to attain the steady choice probability. Simulations of the DFT and EDFT models were replicated for 10,000 times, based on which the choice probability has been calculated. It is noted that the time at which the human subject makes a decision may be different from those of the DFT or EDFT models. For

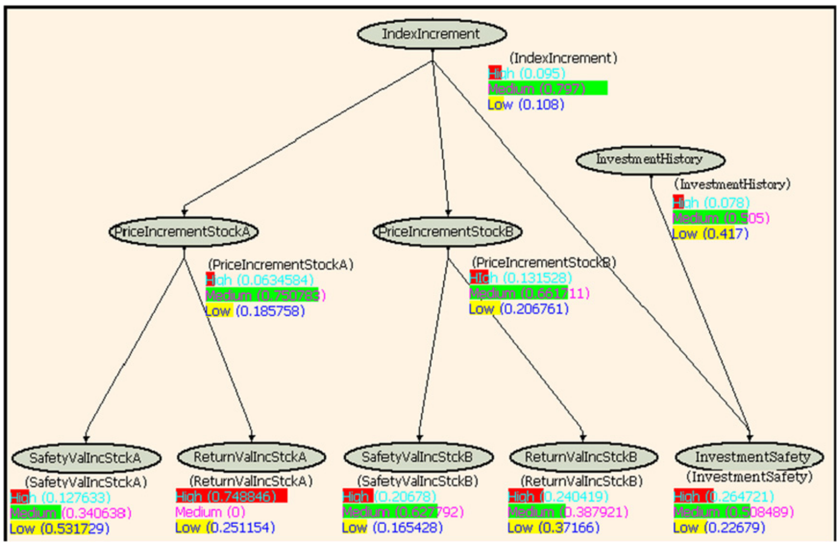

Fig. 11. Bayesian belief network constructed from the stock trading experiment.

Table 3
Conditional probability Pr(Investment safety|Investment history, Index increment)

Parent node(s)		Investment safety		
Investment history	Index increment	High	Medium	Low
High	High	0.2	0.4	0.4
	Medium	0.813	0.187	0.0
	Low	1.0	0.0	0.0
Medium	High	0.125	0.375	0.5
	Medium	0.306	0.625	0.069
	Low	0.383	0.51	0.107
Low	High	0.08	0.32	0.6
	Medium	0.131	0.481	0.388
	Low	0.167	0.533	0.3

the DFT and EDFT simulations, the decision-making time has been obtained from Theorems 3 and 4 (in Section 4) as they provide us with the number of estimated time steps (n) required to obtain the stabilized choice probabilities. For example, Theorems 3 and 4 suggested 1 and 12 as the required time steps for the DFT and EDFT in the first decision, respectively. These required time steps are calculated by Theorems 3 and 4 based on the weight vector $W(t)$ or the value matrix $M(t)$ inferred from the BBN. In the 1st decision of the additional experiment, the human decision-maker changed the weight vector $W(t)$ or the value matrix $M(t)$ 4 times. Then, based on the same information about the stock index and prices when the decision-maker changes $W(t)$ or $M(t)$, the BBN inferred the weight vector $W(t)$ or the value matrix $M(t)$. Using these dynamically changed $W(t)$ and $M(t)$, EDFT evolved until its choice probabilities were stabilized. Fig. 12 depicts the choice probability over n time steps for the first decision of the actual EDFT simulation, where we can observe that the choice probability is actually stabilized at 12 (as suggested by Theorem 4).

Table 4 shows performance of DFT and EDFT in terms of the probability of choosing the same option against the actual choice of the human subject for 10 decisions considered in the experiment. For example, in decisions 4, 6, 8, 9, and 10, DFT predicts an opposite option against the option selected by the subject, but EDFT predicts the same option. In decision 7, however, the EDFT model did not change the matrix

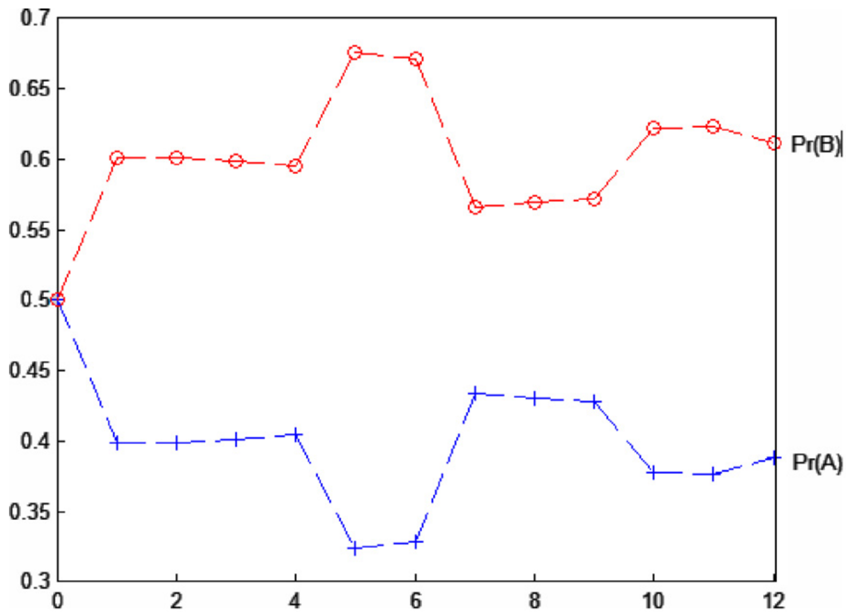

Fig. 12. Stabilization of choice probability over n time steps (1st decision) in EDFT.

Table 4
Choice probabilities of each model in 10 simulated experiment

			Choice probability	Actual choice
Decision 1	DFT	Pr(A)	0.3934	B
		Pr(B)	0.6066	
	EDFT	Pr(A)	0.3724	
		Pr(B)	0.6276	
Decision 2	DFT	Pr(A)	0.4547	B
		Pr(B)	0.5453	
	EDFT	Pr(A)	0.2471	
		Pr(B)	0.7529	
Decision 3	DFT	Pr(A)	0.3995	B
		Pr(B)	0.6005	
	EDFT	Pr(A)	0.4635	
		Pr(B)	0.5365	
Decision 4	DFT	Pr(A)	**0.5984**	B
		Pr(B)	0.4016	
	EDFT	Pr(A)	0.4386	
		Pr(B)	0.5614	
Decision 5	DFT	Pr(A)	0.6124	A
		Pr(B)	0.3876	
	EDFT	Pr(A)	0.8739	
		Pr(B)	0.1261	
Decision 6	DFT	Pr(A)	**0.6534**	B
		Pr(B)	0.3466	
	EDFT	Pr(A)	0.208	
		Pr(B)	0.792	
Decision 7	DFT	Pr(A)	**0.5344**	B
		Pr(B)	0.4656	
	EDFT	Pr(A)	**0.5344**	
		Pr(B)	0.4656	
Decision 8	DFT	Pr(A)	**0.5015**	B
		Pr(B)	0.4985	
	EDFT	Pr(A)	0.2629	
		Pr(B)	0.7371	
Decision 9	DFT	Pr(A)	0.4984	A
		Pr(B)	**0.5016**	
	EDFT	Pr(A)	0.7298	
		Pr(B)	0.2702	
Decision 10	DFT	Pr(A)	0.4991	A
		Pr(B)	**0.5009**	
	EDFT	Pr(A)	0.7801	
		Pr(B)	0.2199	

$M(t)$ nor vector $W(t)$ during the deliberation. So, both DFT and EDFT predicted an incorrect option. The bold numbers in Table 4 denote the case that the model predicts an opposite (incorrect) option compared with the subject. DFT gives 6 such cases out of 10, whereas EDFT gives only one such case. Therefore, we can see that performance of EDFT is better than that of DFT. These results demonstrate the effectiveness of the proposed EDFT under the dynamically changing environment. Fig. 13 depicts the probability of predicting the correct option (same option with the human subject) for DFT (solid line) and EDFT (dotted line) in the considered 10,000 replications of simulations. As shown in Fig. 13, we can see that EDFT gives higher probability to select the correct option in most of the decisions in the experiment. Also in 9 out of 10 cases, it gives the probability that is greater than 0.5. Again, the simulation results validated the accuracy of the proposed EDFT model against the dynamically changing environment.

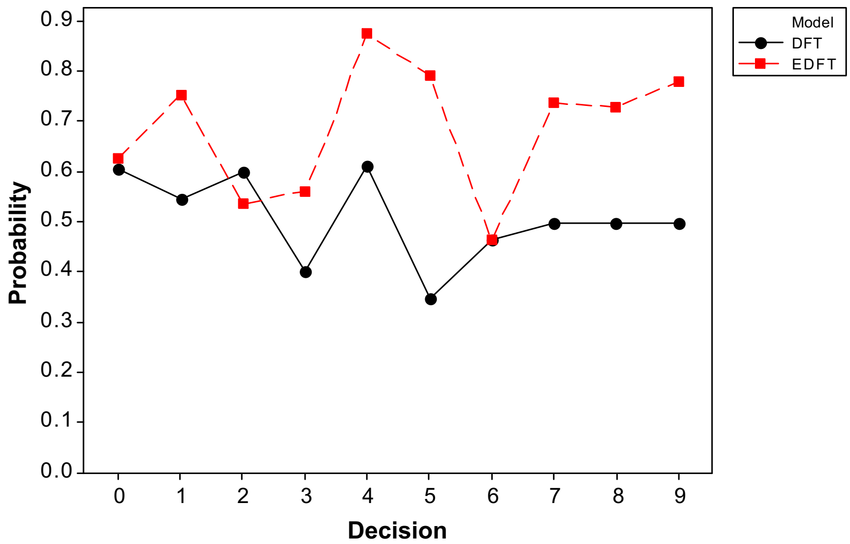

Fig. 13. Probability of predicting the correct option for DFT and EDFT in the considered 10,000 replications of simulations.

6. Conclusions

In this paper, we proposed an extension to Decision field theory (DFT) and presented four corresponding, important theorems. To the best of our knowledge, this is the first effort to extend DFT to cope with the dynamically changing environment. From the Matlab simulations, we demonstrated that the proposed EDFT (extended DFT) could cope with the dynamically changing environment efficiently. In addition, results from the human-in-the-loop experiment showed that EDFT can mimic the human decisions in the dynamically changing environment. In addition, the theorems presented in this paper increase the usability of DFT. By using these theorems, we can obtain the expected preference values without actually deploying the DFT formula. Also we can obtain the desired number of time steps to attain the converged value of the expected preference or the steady choice probability. Since DFT has been built on the psychological principles, it represents psychological behavior of the human, and DFT and EDFT can be applied to many applications involving human behaviors. One of our future works is to apply the proposed EDFT to crowd simulation, where each individual in the crowd is modeled with the EDFT.

Acknowledgement

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