

# Tooling Adjustment Strategy for Acceptable Product Quality in Assembly Processes

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This paper develops an approach to minimize the number of process tooling adjustments and deliver an acceptable fraction of non-conforming products based on given product quality specification limits in assembly processes. A linear model is developed to describe the relationships between product quality and process tooling locating positions. Based on the model, the process mean shifts of tooling locating positions are estimated for both deterministic and stochastic cases by using the least-square estimation or linear mixed model estimation, respectively. A simultaneous confidence interval is obtained to construct the estimation region of a process mean shift under the given false alarm rate. Furthermore, a tooling adjustment strategy is proposed to determine when the process adjustment is essentially needed in order to ensure an acceptable fraction of non-conforming units based on the given product quality specification limits. Finally, a case study is conducted to illustrate the developed methodology by using a real-world autobody assembly process. Copyright © 2010 John Wiley & Sons, Ltd.

**Keywords:** mean shift; simultaneous confidence interval; acceptable fraction non-conforming; linear mixed model; process adjustment

## 1. Introduction

In an automotive assembly process, fixture faults are considered as one major root cause of dimensional variation<sup>1</sup>. Here fixture faults are referred as the significant deviations of locating pins or blocks from their nominal positions relative to the designed part features. There are two kinds of fixture faults: (i) the fault due to the mean shift of fixture locating positions, and (ii) the fault due to the increasing variances of fixture locating positions. In practice, a mean-shift fault is often feasible to be corrected by tooling adjustments. Therefore, this paper will mainly focus on how to monitor process mean shifts and determine when tooling adjustments are essentially needed to satisfy the requirement of acceptable non-conforming units. Generally, the part locating positions, which are determined by the fixture tooling condition, are usually not directly measurable. This brings out research challenges on how to quickly determine fixture mean shifts, and furthermore, how to decide when a tooling adjustment is essentially needed under the given satisfactory fraction of non-conforming products.

Statistical Process Control (SPC) has been widely used in industry. It mainly focuses on detecting process changes and further identifying and eliminating root causes for process improvement. The detailed discussion of SPC is provided by Montgomery<sup>2</sup>. The new concept of integrating SPC with Automatic Process Control (APC) provides an active approach for process variation reduction through compensating process changes by using feedback and/or feedforward control. A detailed discussion on SPC and APC integration can be found in Reference<sup>3</sup>, and a summary and a unified view on the process adjustment problem is provided by Del Castillo *et al.*<sup>4</sup>. Furthermore, a Run by Run (RbR) controller was developed to automatically adjust the process operation parameters to ensure consistency in process outputs<sup>5</sup>. The problem of a dead band in process adjustment was also studied in<sup>6,7</sup> by considering the deterministic drift or shift and their effects of delay in adjustment action on the process operation performance. Recently, Jin and Ding<sup>8</sup> further developed a cautious control method based on a response surface model by considering the measurement uncertainty of process inputs of blank thickness for stamping process control. Although the new concept of integration of SPC and APC has been increasingly applied in many manufacturing industries, there is still limited application in autobody assembly processes due to the numerous varieties of fixtures and the high cost of installing adaptive tooling. In another aspect, when the process capability is high, some small process mean shifts are usually acceptable since the process can still deliver a satisfactory fraction of non-conforming products. In this case, a process adjustment is only needed when the mean shift is large enough. Therefore, it is of interest in determining the acceptable range of mean shifts to avoid

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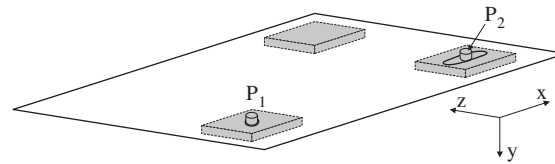


Figure 1. A 3-2-1 fixturing of the rigid part

unnecessary tooling adjustments under the given requirement of product quality specifications and the acceptable fraction of non-conforming units.

Acceptance control chart was first proposed by Freund<sup>9</sup>, who designed the control limits to achieve an acceptable fraction of non-conforming units. This control chart is especially used when the process natural dispersion is small relative to the specification limits such that the process is still capable of producing the satisfactory fraction non-conforming even under a certain mean shift. Wesolowsky<sup>10</sup> proposed the procedure to find a sample size and control limits that minimized the sampling cost while ensuring a fraction of non-conforming units. This acceptance control chart is a simultaneous chart for two correlated processes. Steiner and Wesolowsky<sup>11</sup> extended this work for products with multiple characteristics. Duncan<sup>12</sup> provided a detailed discussion on this topic. As the acceptance control chart allows a small mean shift under the given fraction of non-conforming units, it minimizes the unnecessary alarm signals of out-of-control conditions. In these existing approaches, the derivation of the control charts is directly based on product quality specification limits, which cannot be directly applied for the decision of tooling adjustments because such a decision needs to know which tool and how much the tool adjustments should be done once the out-of-control condition is indicated by the acceptance control chart. Therefore, further research effort is still needed to develop an effective tooling adjustment strategy based on the product quality specifications, which can not only ensure product quality under an acceptable fraction of non-conforming units, but also avoid unnecessary tooling adjustments.

The objective of this paper is to develop an effective tooling adjustment strategy for assembly processes based on the given product quality specification limits and the acceptable fraction of non-conforming products, which can avoid unnecessary tooling adjustments. Specifically, this paper includes three major steps for the methodology development: (a) modeling of the relationship between the product quality and process fixture locating positions and transforming the product quality specification limits into tooling tolerance region; (b) estimation of the mean shift of fixture locating position deviations based on the product measurements; and (c) development of a tooling adjustment strategy by considering the estimation uncertainty of tooling deviations. The remainder of the paper is organized as follows. Section 2 discusses the detailed development of the proposed methodology. In Section 3, a case study is performed using an exemplary autobody assembly process. Finally, the paper is concluded in Section 4.

## 2. Methodology development

Before discussing the methodology development, a fixture tooling system and its associated faults are introduced first. In this research, a commonly used 3-2-1 fixturing system for rigid part assembly is considered. As shown in Figure 1, there are two locating pins:  $P_1$  and  $P_2$ .  $P_1$  is a four-way pin, which has a pin-hole configuration to control the part position in both the  $x$ - and  $z$ -directions.  $P_2$  is a two-way pin that has a pin-slot configuration to control the part position only in the  $z$ -direction. Three blocks are used to control the part position in the  $y$ -direction.

In this study, the mean shifts of locators are investigated. In practice, a mean shift may be caused by a block wear out or loosen, a bent pin or a mislocated pin, etc. These faults have a linear relationship with the geometry of the part orientations, thus leading to assembled part dimensional deviation or quality defects. The first effort of the methodology development is to model the relationships between mean shifts of locating positions and the part quality.

### 2.1. Modeling of relationships between part quality and tooling mean shifts

With the assumption of small locating position deviations, a linear model can be obtained to represent the relationships between part quality and fixture locating position deviations for assembly processes<sup>13-16</sup>:

$$\mathbf{y}^i = \mathbf{C}\mathbf{u}^i + \mathbf{w}^i, \quad i = 1, 2, \dots, n. \quad (1)$$

Here,  $\mathbf{y}^i$  denotes an  $m$ -dimensional measurement vector on the  $i$ th part;  $\mathbf{C}$  denotes an  $m \times p$  linearized effect matrix of each tooling deviation on the measurement vector  $\mathbf{y}^i$ , obtained by kinematics analysis;  $\mathbf{u}^i$  denotes a  $p$ -dimensional tooling deviation vector that has potential mean shifts;  $\mathbf{w}^i$  denotes the  $m$ -dimensional vector of independent and identical measurement noises at individual measurement points on products, which follow the multivariate normal distribution of  $\mathbf{w}^i \sim N(\mathbf{0}, \text{diag}\{\sigma_w^2 \dots \sigma_w^2\})$  with the same variance  $\sigma_w^2$  for different measurement points. The value of  $\sigma_w^2$  can be obtained based on the sensor specifications from sensor manufacturers or estimated based on the existing technique of gage R&R study<sup>2</sup>.

The tooling deviation vector,  $\mathbf{u}^i$ , can be decomposed into a fixed component and a random component as:

$$\mathbf{u}^i = \boldsymbol{\mu} + \mathbf{v}^i, \quad (2)$$

Here,  $\boldsymbol{\mu}$  is a  $p$ -dimensional mean vector of each fault.  $\mathbf{v}^i$  is a  $p$ -dimensional random vector representing the random variation of each fault component, which indicates that individual parts can be randomly located within the design tolerances of the clearances between parts and fixture tooling. Under the assumption of independent fixture faults,  $\mathbf{v}^i$  is assumed to follow the normal distribution of  $\mathbf{v}^i \sim N(\mathbf{0}, \text{diag}\{\sigma_{u_j}^2\}_{j=1, \dots, p})$ , where  $\sigma_{u_j}^2$  denotes the variance of individual locator component  $j$ , which can be obtained based on the designed process tolerance<sup>17</sup>. Therefore, a linear model with potential mean shifts is expressed as follows:

$$\mathbf{y}^i = \mathbf{C}\boldsymbol{\mu} + (\mathbf{C}\mathbf{v}^i + \mathbf{w}^i), \quad i = 1, 2, \dots, n, \quad (3)$$

$$\begin{bmatrix} \mathbf{y}^1 \\ \mathbf{y}^2 \\ \vdots \\ \mathbf{y}^n \end{bmatrix}_{nm} = \begin{bmatrix} \mathbf{C}_{(m \times p)} \\ \mathbf{C}_{(m \times p)} \\ \vdots \\ \mathbf{C}_{(m \times p)} \end{bmatrix}_{(nm) \times p} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix}_p + \begin{bmatrix} \mathbf{C}\mathbf{v}^1 + \mathbf{w}^1 \\ \mathbf{C}\mathbf{v}^2 + \mathbf{w}^2 \\ \vdots \\ \mathbf{C}\mathbf{v}^n + \mathbf{w}^n \end{bmatrix}_{nm}$$

$\mathbf{y}$                        $\boldsymbol{\Gamma}$                        $\boldsymbol{\mu}$                        $\boldsymbol{\varepsilon}$

Or

$$\mathbf{y} = \boldsymbol{\Gamma}\boldsymbol{\mu} + \boldsymbol{\varepsilon}. \quad (4)$$

Equation (4) will be used to estimate the mean deviations of the process faults.

## 2.2. Estimation of mean shifts

In this section, the mean shifts will be estimated by using the linear model (4). In practice, there are two types of mean-shift faults: (i) non-random mean-shift faults contributed by bent pins, mislocated pins, or broken pins, etc. Under such extreme tooling faults, the excessive mean shifts of fixture locating positions are generated, which affect the product quality much more dominantly than the random error  $\mathbf{v}^i$  due to the designed fixture tolerance; and (ii) random mean-shift faults contributed by the loosened or worn-out blocks or locating pins, etc. Under such tooling faults, mean shifts show a random variation pattern, which is affected by the design tolerance of the clearance between parts and fixture tooling. Different algorithms will be adapted to estimate these two types of mean-shift faults, respectively.

(1) For non-random tooling deviations,  $\mathbf{v}^i = \mathbf{0}, \forall i$

As  $\mathbf{v}^i = \mathbf{0}, \forall i$ , the model (3) becomes  $\mathbf{y}^i = \mathbf{C}\boldsymbol{\mu} + \mathbf{w}^i$ .  $\boldsymbol{\mu}$  can be estimated by using the least-squares estimation method, i.e.

$$\hat{\boldsymbol{\mu}} = (\boldsymbol{\Gamma}^T \boldsymbol{\Gamma})^{-1} \boldsymbol{\Gamma}^T \mathbf{y} = \frac{1}{n} (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T \sum_{i=1}^n \mathbf{y}^i, \quad (5)$$

$$\boldsymbol{\Sigma}_{\hat{\boldsymbol{\mu}}} = \text{var}(\hat{\boldsymbol{\mu}}) = \sigma_w^2 (\boldsymbol{\Gamma}^T \boldsymbol{\Gamma})^{-1} = \frac{1}{n} \sigma_w^2 (\mathbf{C}^T \mathbf{C})^{-1}, \quad (6)$$

where (5) provides the estimated mean shifts, and (6) provides the covariance of the estimator that reflects the uncertainty of the estimation.

(2) For random tooling deviations,  $\mathbf{v}^i \neq \mathbf{0}$

As  $\mathbf{v}^i \neq \mathbf{0}$ , assume  $\mathbf{v}^i \sim N(\mathbf{0}, \text{diag}\{\sigma_{u_j}^2\}_{j=1, \dots, p})$ , the random part in the model is the summation of the vectors of  $\mathbf{C}\mathbf{v}^i + \mathbf{w}^i$ , which is denoted as

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \mathbf{C}\mathbf{v}^1 + \mathbf{w}^1 \\ \vdots \\ \mathbf{C}\mathbf{v}^n + \mathbf{w}^n \end{bmatrix}_{(nm) \times 1}$$

As a result, the covariance of the random term is no longer diagonal. In this study, the variances ( $\sigma_{u_j}^2$  and  $\sigma_w^2$ ) are assumed constant and known, which can be obtained from the estimation based on historical data, the specifications of the tooling elements, tolerances of the part, and sensor specifications.

For the case with a non-diagonal variance of  $\mathbf{y}$ , a linear mixed model is used to estimate its covariance. Assume that there are no interactions among the locator deviations, then the covariance matrix of  $\mathbf{u}$ ,  $\boldsymbol{\Sigma}_{\mathbf{u}}$ , is diagonal, i.e.  $\boldsymbol{\Sigma}_{\mathbf{u}} = \text{diag}[\sigma_{u_1}^2, \dots, \sigma_{u_p}^2]$ . By taking covariance on both sides of (4), we have

$$\boldsymbol{\Sigma}_{\mathbf{y}} = \text{cov}(\mathbf{y}) = \text{cov}(\boldsymbol{\Gamma}\boldsymbol{\mu} + \boldsymbol{\varepsilon}) = \text{var}(\boldsymbol{\varepsilon}) = \begin{bmatrix} \mathbf{C}\boldsymbol{\Sigma}_{\mathbf{u}}\mathbf{C}^T + \sigma_w^2 \mathbf{I} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{C}\boldsymbol{\Sigma}_{\mathbf{u}}\mathbf{C}^T + \sigma_w^2 \mathbf{I} \end{bmatrix}_{nm \times nm}. \quad (7)$$

It is known that  $\mathbf{y} \sim N(\Gamma\boldsymbol{\mu}, \Sigma_{\mathbf{y}})$ . The ordinary least-square estimation provides unbiased linear estimates. Although the covariance of  $\mathbf{y}$  is not diagonal, the ordinary least-squares estimation can still be used to provide an unbiased estimate of each mean fault. However, these estimates do not have the minimum variance property<sup>18</sup>. The analysis based on a linear mixed model with fixed effects and known variance can be used to estimate the mean faults and also provide the minimum variance for the estimate. In this paper, a linear mixed model is used to get the following estimation results:

$$\hat{\boldsymbol{\mu}} = (\Gamma^T \Sigma_{\mathbf{y}}^{-1} \Gamma)^{-1} \Gamma^T \Sigma_{\mathbf{y}}^{-1} \mathbf{y}$$

$$= \left( \sum_{i=1}^n \mathbf{C}^T (\mathbf{C} \Sigma_{\mathbf{u}} \mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} \mathbf{C} \right)^{-1} \left( \sum_{i=1}^n \mathbf{C}^T (\mathbf{C} \Sigma_{\mathbf{u}} \mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} \mathbf{y}^i \right) \quad (8)$$

$$\Sigma_{\hat{\boldsymbol{\mu}}} = \text{var}(\hat{\boldsymbol{\mu}}) = (\Gamma^T \Sigma_{\mathbf{y}}^{-1} \Gamma)^{-1} = \left( \sum_{i=1}^n \mathbf{C}^T (\mathbf{C} \Sigma_{\mathbf{u}} \mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} \mathbf{C} \right)^{-1}. \quad (9)$$

Appendix A provides the detailed derivations for the results in (5)–(9).

### 2.3. Simultaneous confidence intervals of mean shifts

In the paper, simultaneous confidence intervals, rather than an individual interval for each mean shift, is used to construct a tooling estimation region to avoid the excessive number of false alarms. There are various methods available to construct the simultaneous confidence intervals. One common approach is the Bonferroni method using the individual false alarm rates  $\alpha_j = \alpha/p$ , where  $\alpha$  is the designed ideal total false alarm rate. However, the actual total false alarm error tends to be much smaller than  $\alpha$  due to the correlation between variables. Another approach is Hotelling's statistic, which approximately maintains a total false alarm error rate equal to a specified level  $\alpha$ <sup>19</sup>. However, its drawback is that when the process is out of control, the specific variables contributing to this out-of-control signal are not directly known. Hayter and Tsui<sup>20</sup> propose a procedure that provided simultaneous confidence intervals with exact total false alarm rate  $\alpha$  based on  $M$  Statistic. The advantage of this approach is that it is easy to identify out-of-control variables and also satisfy the exact overall false alarm rate  $\alpha$ . This paper adopts this approach to develop a simultaneous confidence interval of the mean-shift estimation, which will be discussed in detail as follows.

Suppose  $\hat{\boldsymbol{\mu}} \sim N_p(\boldsymbol{\mu}, \Sigma_{\hat{\boldsymbol{\mu}}})$ , where  $\Sigma_{\hat{\boldsymbol{\mu}}}$  is a  $p \times p$  variance matrix with the diagonal elements of  $\Sigma_{\hat{\boldsymbol{\mu}}}$  equal to  $\sigma_j^2$  and off-diagonal elements of  $\Sigma_{\hat{\boldsymbol{\mu}}}$  equal to  $\sigma_{jk}$ , for  $j \neq k$ . Suppose  $\mathbf{R}$  is a correlation matrix transformed from  $\Sigma_{\hat{\boldsymbol{\mu}}}$  through  $\rho_{jk} = \sigma_{jk} / (\sigma_j \sigma_k)$ . Let  $D_{\mathbf{R}, \alpha}$  be the critical point at level  $\alpha$ . Without loss of generality of the methodology development, the null hypothesis is  $H_0: \boldsymbol{\mu} = \boldsymbol{\mu}_0$ , where  $\boldsymbol{\mu}_0$  is a zero vector. The alternative hypothesis is  $H_1: \boldsymbol{\mu} \neq \boldsymbol{\mu}_0$ . According to Reference<sup>20</sup>, it yields

$$\Pr\{\mu_j \in [\mu_j - \sigma_j D_{\mathbf{R}, \alpha}, \mu_j + \sigma_j D_{\mathbf{R}, \alpha}] \text{ for } 1 \leq j \leq p\} = 1 - \alpha.$$

That is, the simultaneous confidence intervals with confidence level of  $1 - \alpha$  are  $[\mu_j - \sigma_j D_{\mathbf{R}, \alpha}, \mu_j + \sigma_j D_{\mathbf{R}, \alpha}]$  for  $1 \leq j \leq p$ . Equivalently,  $M$  statistic will conclude a mean shift when the following condition is held:

$$M = \max_{1 \leq j \leq p} \frac{|\hat{\mu}_j - \mu_{0j}|}{\sigma_j} > D_{\mathbf{R}, \alpha}. \quad (10)$$

The following procedure will be used to select an appropriate critical point<sup>20</sup>:

1. A large number ( $N$ ) of the realization vectors  $\boldsymbol{\mu}^1, \dots, \boldsymbol{\mu}^N$  are generated from the  $p$ -dimensional multivariate normal distribution with mean zero and the variance matrix equal to the correlation matrix  $\mathbf{R}$ . Each  $\boldsymbol{\mu}^j$  is  $p$ -dimensional vector.
2.  $M$  statistics of each  $\boldsymbol{\mu}^k = (\mu_1^k, \dots, \mu_p^k)$  are calculated. In the case of  $\boldsymbol{\mu}_0 = \mathbf{0}$  and  $\sigma_j = 1$ , it has

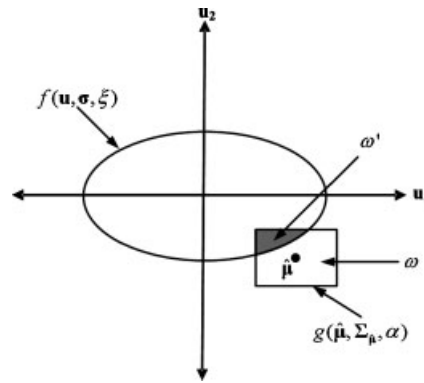
$$M^k = \max_{1 \leq j \leq p} |\mu_j^k| \quad \text{for } k = 1, \dots, N.$$

3. The  $(1 - \alpha)$ th percentile of the set of  $\{M^1, \dots, M^N\}$  is determined and used as an estimate of the critical point  $D_{\mathbf{R}, \alpha}$ .

### 2.4. Simultaneous control strategy under a satisfactory fraction of non-conforming units

After estimating the mean-shift faults, it is desirable to systematically identify when the faults should be corrected through tooling adjustments. In this section, a simultaneous control strategy under a satisfactory fraction of non-conforming units is proposed based on the transformed tooling tolerance range and simultaneous confidence intervals.

It is known that the quality conformance of a product is often assessed by whether its key product characteristics (KPCs) can meet the product specification limits. KPCs are often selected as the critical features of products, which have direct impacts on the customer satisfaction based on the designed product functionality. In the paper, KPCs refer to the critical dimensional measurement points specified on parts; and a non-conforming product is defined by at least one of its KPC measurements exceeding the product specification limits. Moreover, the fraction of non-conforming units is assessed by 100% inspection of every individual product in the paper. We also assumed that measurement gages are capable, and both the false alarm rate and



**Figure 2.** Tolerance and simultaneous confidence regions of two tooling deviations

the miss detection rate regarding the conformity decisions of products' quality due to the measurement gages are ignorable in the paper.

In assembly processes, the dimensional errors of KPCs are essentially affected by key control characteristics (KCCs) of tooling components, which refers to the key positioning elements of fixture locators in the paper, such as the positions of locating pins or locating blocks. Therefore, it is very critical to design appropriate tooling tolerance limits for individual KCCs in order to ensure that the produced products satisfy the acceptable fraction non-conforming  $\xi$ . The paper<sup>21</sup> showed how to transfer the pre-given *product* specification limits of KPCs into the *tooling* tolerance limits of KCCs for assembly processes. It, in fact, provided a conservative design to ensure that the designed tooling system can produce quality products with at most the fraction of non-conforming units equal to  $\xi$  as long as the tooling KPCs' deviations are within the designed tooling tolerance limits. For example, the tooling tolerance region, which is formed by two KCCs' deviations of  $u_1$  and  $u_2$  with an ellipse boundary in Figure 2, is obtained by transferring the product specification limits on KPCs into the tooling tolerance region on KCCs. The detail procedures for how to obtain the ellipse function are given in Reference<sup>21</sup>, which is also summarized in Appendix B.

In practice, tooling locator deviations of KCCs are not directly measurable during production. The following method is proposed to decide whether the actual KCCs deviations during production are within the ellipse of the tooling tolerance region based on KPCs measurements. Suppose  $f(\mathbf{u}, \sigma, \xi)$  is a function representing the ellipse, where  $\mathbf{u}$  is a vector of KCCs deviation and  $\sigma$  is the allocated variances of the allowable tolerances for the KCCs.  $g(\hat{\boldsymbol{\mu}}, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\mu}}}, \alpha)$  is a function representing the simultaneous confidence intervals represented by the rectangle as shown in Figure 2, which is determined using the method in Section 2.3. It should be notified that  $\alpha$  in  $g(\hat{\boldsymbol{\mu}}, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\mu}}}, \alpha)$  is used to represent the confidence level  $(1 - \alpha)$  of the simultaneous confidence intervals of the tooling KCCs' mean shifts by considering the estimation uncertainty. In Figure 2, the rectangle area  $\omega$  is called the simultaneous confidence region, and  $\omega'$  is defined as the intersection area between the simultaneous confidence region (rectangle area  $\omega$ ) and the tooling tolerance region (ellipse area). Therefore,  $\gamma = \omega' / \omega$  represents the fraction of the intersection area over the total rectangle area. Intuitively, it can be understood that a tooling adjustment is not necessarily needed if  $\gamma = 100\%$ , whereas a tooling adjustment is essentially needed if  $\gamma = 0\%$ . Therefore, under an appropriate threshold  $\eta$ , a tooling adjustment is needed whenever the condition of  $\gamma < \eta$  is held. The general strategy to determine whether a tooling adjustment should be done is given as follows:

*Step 1.* Transform the product KPC specification limits into the tooling KCC tolerance using the algorithm provided by Shiu *et al.*<sup>21</sup>, i.e. obtain the tooling tolerance region represented by  $f(\mathbf{u}, \sigma, \xi)$  under the given acceptable fraction of non-conforming products  $\xi$ .

*Step 2.* Estimate the means of tooling deviations,  $\hat{\boldsymbol{\mu}}$ , and obtain the associated covariance of the mean estimates,  $\boldsymbol{\Sigma}_{\hat{\boldsymbol{\mu}}}$ . Construct the simultaneous confidence region represented by  $g(\hat{\boldsymbol{\mu}}, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\mu}}}, \alpha)$ .

*Step 3.* Determine  $\omega'$  and  $\omega$  using the following equations:

$$\omega' = \int_{\Omega} h_{\hat{\boldsymbol{\mu}}}(\mathbf{u}) \, d\mathbf{u}, \quad (11)$$

and

$$\omega = \int_{\Theta} h_{\hat{\boldsymbol{\mu}}}(\mathbf{u}) \, d\mathbf{u} \quad (12)$$

where  $h(\mathbf{u}) \sim N(\hat{\boldsymbol{\mu}}, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\mu}}})$ ,  $\Omega = \{\mathbf{u}: [\mathbf{u} \in g(\hat{\boldsymbol{\mu}}, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\mu}}}, \alpha)] \cap [\mathbf{u} \in f(\mathbf{u}, \sigma, \xi) \leq 0]\}$ , and  $\Theta$  is the set of limits of the simultaneous confidence intervals for each axis. These integrals can be solved using the Monte Carlo simulations. Then, we can get  $\gamma = \omega' / \omega$ .

*Step 4.* If  $\gamma < \eta$ , perform process corrective action via the tooling adjustment. Otherwise, do nothing until the next inspection, and then return to Step 1.

The determination of  $\eta$  can be obtained by simulating  $M$  samples of the mean shifts  $\boldsymbol{\mu}^1, \dots, \boldsymbol{\mu}^M$ . Each  $\boldsymbol{\mu}^j (j=1, \dots, M)$  is a  $p$ -dimensional vector that can satisfy the quality requirement with the fraction of non-conforming units equal to  $\xi$ . Then, we can

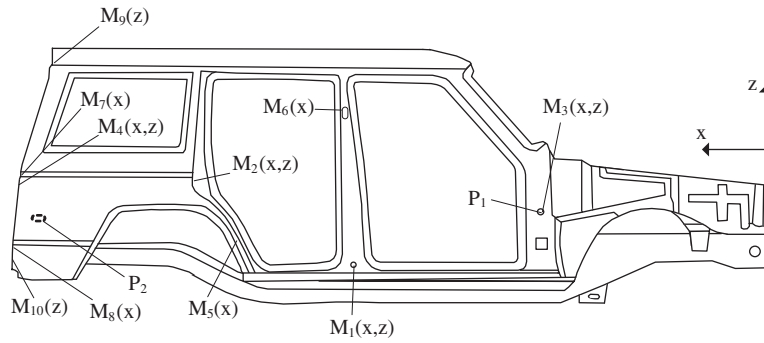


Figure 3. The right-hand aperture of an autobody<sup>13</sup>

Point	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	M <sub>6</sub>	M <sub>7</sub>	M <sub>8</sub>	M <sub>9</sub>	M <sub>10</sub>	P <sub>1</sub>	P <sub>2</sub>
Nominal <b>x</b>	3134	4015	2184	4895.3	3721	3264	4895	4895.5	4693.8	4899	2184	4680
(mm) <b>z</b>	1200	1618.5	1489	1510.5	1256.5	1930	1608	1273	2228.5	1214.5	1489	1428

further get the estimated mean shifts and the simultaneous confidence intervals corresponding to each sample  $\mu^j (j=1, \dots, M)$ , which are further used to calculate  $\gamma^j = \omega^j / \omega^j$  for each sample  $\mu^j$  based on (11) and (12). The conservative threshold  $\eta$  that can ensure the satisfactory quality can be determined by  $\eta = \max(\gamma^j, \forall j=1, \dots, M)$ . The detailed simulation procedures via a grid searching approach are given as follows:

*Step 1:* Find the feasible maximum range for each tooling component’s deviation. In this step, each tooling component is investigated individually. Specifically, in order to obtain the maximum range for  $u_i (i=1, \dots, p)$ , we set all other tooling components’ deviations equal to zero. The maximum range of  $u_i$  is obtained by iteratively increasing  $u_i$  with a small searching step until the constraint of the given fraction of non-conforming units is not held.

*Step 2:* Use a grid searching approach to obtain the feasible tooling deviations by considering all combinations of tooling components’ deviations. Specifically, within these maximum ranges of individual tooling components obtained in Step 1, we simulate the tooling component deviations with each increasing step of  $\Delta = \text{maximum range} / N_0$  ( $N_0$  is the total number of simulation steps for each KCC,  $N_0 = 100$  is used in the paper). Those samples with the feasible tooling deviation boundary are selected to ensure the fraction of non-conforming units equal to  $\xi$ . The total number of the selected samples in this step is denoted as  $M$ .

*Step 3:* Calculate  $\gamma_j (j=1, \dots, M)$  for each sample of the selected feasible tooling deviations, which is obtained based on (11) and (12) using the estimated mean shifts and simultaneous confidence intervals.

*Step 4:* Determine the threshold  $\eta$ , which is equal to the maximum of  $\gamma_j (j=1, \dots, M)$ .

### 3. Case study

A case study is conducted in an automotive body assembly application to illustrate the developed methodology. The part used in the case study is a side aperture of a car body as shown in Figure 3, in which two locating pins, i.e.  $P_1$  as the four-way pin and  $P_2$  as the two-way pin, are used. In this case study, only the variation in the x–z plane is considered for the side aperture assembly. There are 10 points being measured along the x- and z-directions. Table I provides the nominal positions in the x–z coordinates of those pins and measurement points.

In this case study, for each part  $i$ , the measurement  $\mathbf{y}^j$  is a four-dimensional vector,  $\mathbf{y}^j \equiv [M_2(x) \ M_8(x) \ M_1(z) \ M_9(z)]^T$ . Therefore,  $m=4$ . In this case study, two fixture faults are considered, which are  $P_1$  failure in the x-direction (fault 1) and  $P_2$  failure in the z-direction (fault 2), i.e.  $p=2$ . The  $\mathbf{C}$  matrix in Equation (1) is<sup>13</sup>:

$$\mathbf{C}^T = \begin{bmatrix} 0.354 & 0.354 & 0 & 0 \\ -0.026 & 0.043 & 0.187 & 0.495 \end{bmatrix}$$

The model parameters used in this study are listed in Table II.

$n$	$\sigma_w$	$\sigma_{u_1}$	$\sigma_{u_2}$	$\alpha$	$\xi$	$\eta$	$\{T_{y_j}\}_{j=1}^m$
50	0.1	0.5	0.5	0.05	0.05	0.95	1.0 mm

Case	$\hat{\mu}_1$	$\hat{\mu}_2$	$\Sigma_{\hat{\mu}}(10^{-3})$	$D_{R,x}$	S. I. of $\mu_1$	S. I. of $\mu_2$
$\mathbf{v}^j = \mathbf{0}$	0.484	2.040	$\begin{bmatrix} 0.799 & -0.019 \\ -0.019 & 0.709 \end{bmatrix}$	2.243	[0.421, 0.528]	[1.978, 2.099]
$\mathbf{v}^j \neq \mathbf{0}$	0.443	1.959	$\begin{bmatrix} 5.8 & -0.000 \\ -0.000 & 5.7 \end{bmatrix}$	2.237	[0.273, 0.614]	[1.790, 2.128]

\*S. I. stands for simultaneous interval.

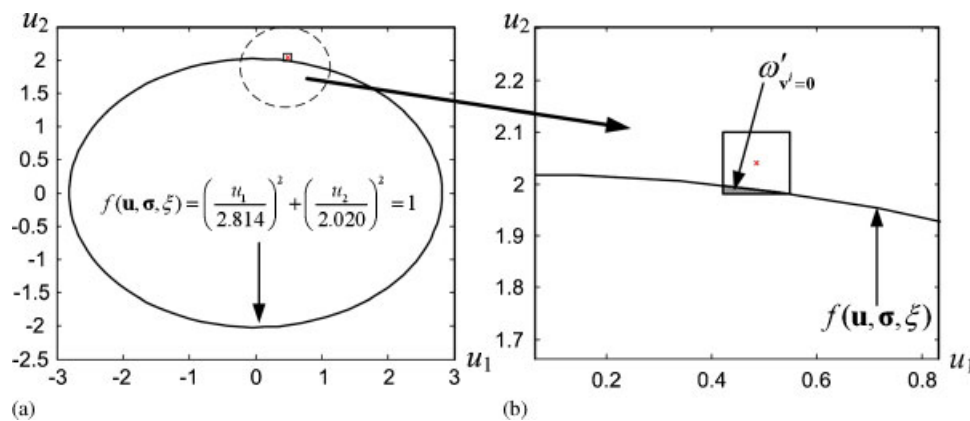


Figure 4. The KCC tolerance region and simultaneous confidence region for  $\mathbf{v}^j = \mathbf{0}$ : (a) KCC tolerance region and simultaneous confidence region and (b) Intersection areas for both cases

### 3.1. Mean estimations and simultaneous confidence intervals

Two case studies were conducted: the first case assumes  $\mathbf{v}^j = \mathbf{0}$  and the second case assumes  $\mathbf{v}^j \neq \mathbf{0}$ . In both cases, the mean-shift faults of 0.5 mm and 2.0 mm are generated from the tooling error of  $P_1$  in the x-direction and tooling error of  $P_2$  in the z-direction, respectively. The results of the simultaneous tests are provided for each case as shown in Table III.

### 3.2. Simultaneous control strategy

The simultaneous control strategy under a satisfactory fraction of non-conforming units is performed for two cases of  $\mathbf{v}^j = \mathbf{0}$  and  $\mathbf{v}^j \neq \mathbf{0}$ . The KPC tolerances,  $\{T_{y_j}\}_{j=1}^m = 1.0\text{mm}$ , are transformed into the KCC tolerances according to Reference<sup>21</sup>. From Appendix B, the function of this ellipse is

$$f(\mathbf{u}, \sigma, \xi) = \left(\frac{u_1}{2.814}\right)^2 + \left(\frac{u_2}{2.020}\right)^2 = 1,$$

where  $\sigma = [1.150^2 \ 0.825^2]^T$  and the acceptable fraction of non-conforming  $\xi = 5\%$ . The estimated  $\hat{\mu}$  and simultaneous confidence intervals determined in the previous section are investigated using the procedure in Section 2.4 to determine when to perform the corrective action under the given fraction of non-conforming units at most  $\xi = 5\%$ . The simulation shows that the conservative threshold  $\eta$  is 0.70 for  $\mathbf{v}^j = \mathbf{0}$  and 0.95 for  $\mathbf{v}^j \neq \mathbf{0}$ .

Under the given mean-shift faults, Figures 4 and 5 show the tolerance region, the simultaneous confidence region, and their intersection areas of the tooling deviation corresponding to  $\mathbf{v}^j = \mathbf{0}$  and  $\mathbf{v}^j \neq \mathbf{0}$ , respectively.  $\omega'_{\mathbf{v}^j = \mathbf{0}}$  and  $\omega'_{\mathbf{v}^j \neq \mathbf{0}}$  are the intersection areas between the simultaneous confidence region and the tolerance region for  $\mathbf{v}^j = \mathbf{0}$  and  $\mathbf{v}^j \neq \mathbf{0}$ , respectively.  $\omega_{\mathbf{v}^j = \mathbf{0}}$  and  $\omega_{\mathbf{v}^j \neq \mathbf{0}}$  are the areas of simultaneous confidence region for  $\mathbf{v}^j = \mathbf{0}$  and  $\mathbf{v}^j \neq \mathbf{0}$ , respectively.

In Figure 4(a), the rectangle is the simultaneous confidence region and the 'x' is the position of the  $\hat{\mu}$  under  $\mathbf{v}^j = \mathbf{0}$ . In Figure 5(a), the rectangle is the simultaneous confidence region and the '+' is the position of the  $\hat{\mu}$  under  $\mathbf{v}^j \neq \mathbf{0}$ . The dash circle is enlarged and presented in Figures 4(b) and 5(b), respectively. From (11) and (12) and given the  $h_{\hat{\mu}}(\mathbf{u})$  for  $\mathbf{v}^j = \mathbf{0}$  and  $\mathbf{v}^j \neq \mathbf{0}$ , the

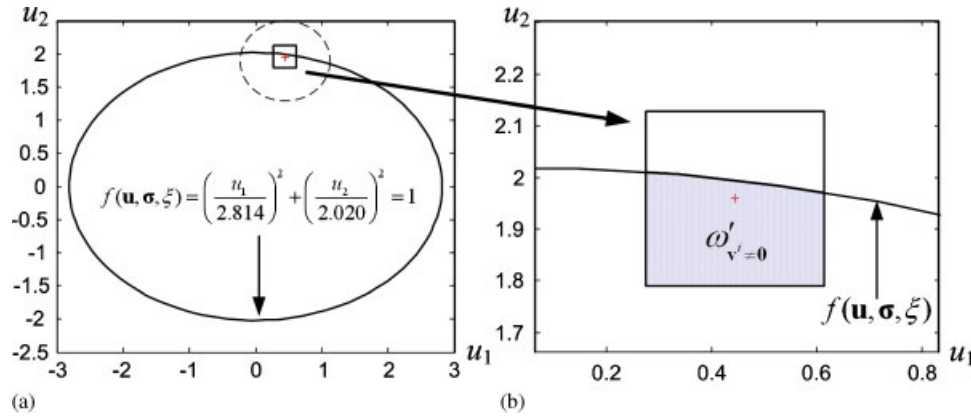


Figure 5. The KCC tolerance region and simultaneous confidence region for  $\mathbf{v}^j = \mathbf{0}$ : (a) KCC tolerance region and simultaneous confidence region and (b) Intersection areas for both cases

Table IV. The results from the simultaneous control strategy				
Case	$\omega'$	$\omega$	$\gamma$	% non-conforming unit
$\mathbf{v}^j = \mathbf{0}$	0.0799	0.9487	0.0842	0.11
$\mathbf{v}^j \neq \mathbf{0}$	0.5694	0.9427	0.6040	0.14

Monte Carlo simulation is used to determine the areas of  $\omega'_{\mathbf{v}^j = \mathbf{0}}$ ,  $\omega'_{\mathbf{v}^j \neq \mathbf{0}}$ ,  $\omega_{\mathbf{v}^j = \mathbf{0}}$ , and  $\omega_{\mathbf{v}^j \neq \mathbf{0}}$  in Figures 4(b) and 5(b), respectively. The results are shown in Table IV.

From Table IV, the corrective action needs to be performed for both cases since the  $\gamma$ 's are less than the threshold values of  $\eta$  of 0.70 and 0.95 corresponding to  $\mathbf{v}^j = \mathbf{0}$  and  $\mathbf{v}^j \neq \mathbf{0}$ , respectively. Although, the estimated  $\hat{\mu}$  under  $\mathbf{v}^j \neq \mathbf{0}$  is located inside the tolerance ellipse in Figure 5(b), the result of  $\gamma$  shows that this set of the tooling deviations produces more than 5% of non-conforming units, which is not acceptable. In addition, the fractions of non-conforming units are higher than 5% for both cases, which confirms the need for the corrective action.

#### 4. Summary

In this paper, a new methodology was developed to determine when tooling adjustments are essentially needed based on product quality specification limits and the acceptable fraction of non-conforming products. In the paper, a linear fault model is proposed to determine the geometric relationship between part quality measurements and fixture locating position deviations. Based on this linear model, a least-square estimation and a linear mixed model are used to estimate the potential mean-shift faults and assess the variance of these estimations. A simultaneous confidence interval is constructed to determine the mean-shift estimation region under the given false alarm rate. Furthermore, by considering the estimation uncertainty, a decision strategy is proposed to determine when a tooling adjustment is essentially needed for ensuring the acceptable fraction of non-conforming units and meanwhile for avoiding unnecessary tooling adjustments. A case study using an autobody assembly process was conducted to illustrate the implementation procedures and the effectiveness of the developed methodology.

The proposed methodology can also be applied to a general multi-station assembly process. It is known that a state-space model has been widely used for effectively modeling the variation propagations in a multi-station assembly process<sup>14</sup>, which can be further transferred into a linear regression model that is used in the paper. The detailed relationship between a linear regression model and a state-space model for a multistage manufacturing process and a linear regression model can also be found in Reference<sup>22</sup>.

Furthermore, it should be notified that the proposed strategy will bring forth direct savings in the production cost by avoiding unnecessary tooling adjustments. Extensive future research is still needed to determine the amount of adjustments of individual tools by considering the production cost associated with each tool adjustment. For this purpose, an optimal objective function needs to be defined for achieving an optimal tooling adjustment strategy, such as minimizing the total number of adjustments over the production cycle<sup>23</sup>, and minimizing the total production variations<sup>8</sup>, and minimizing the number of resetting tooling components<sup>24</sup>.

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## Appendix A: Mean faults estimation

This appendix provides the derivations of  $\hat{\boldsymbol{\mu}}$  and  $\Sigma_{\hat{\boldsymbol{\mu}}}$  for each case.

- (1) For non-random tooling deviations,  $\mathbf{v}^i = \mathbf{0}, \forall i$ . The least-square estimation is applied to the linear model in (4) and the results of the estimation are expressed as follows:

The estimated mean shift,  $\hat{\boldsymbol{\mu}}$  is:

$$\begin{aligned} \hat{\boldsymbol{\mu}} &= (\mathbf{\Gamma}^T \mathbf{\Gamma})^{-1} \mathbf{\Gamma}^T \mathbf{y} \\ &= \left( \begin{bmatrix} \mathbf{c}^T & \dots & \mathbf{c}^T \\ \vdots \\ \mathbf{c} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{c}^T & \dots & \mathbf{c}^T \\ \vdots \\ \mathbf{c} \end{bmatrix} \begin{bmatrix} \mathbf{y}^1 \\ \vdots \\ \mathbf{y}^n \end{bmatrix} \\ &= \left( \sum_{i=1}^n \mathbf{c}^T \mathbf{c} \right)^{-1} \mathbf{c}^T \sum_{i=1}^n \mathbf{y}^i = (n \mathbf{c}^T \mathbf{c})^{-1} \mathbf{c}^T \sum_{i=1}^n \mathbf{y}^i = \frac{1}{n} (\mathbf{c}^T \mathbf{c})^{-1} \mathbf{c}^T \sum_{i=1}^n \mathbf{y}^i. \end{aligned} \quad (\text{A1})$$

The covariance of the estimated mean shift,  $\Sigma_{\hat{\boldsymbol{\mu}}}$ , is:

$$\begin{aligned} \Sigma_{\hat{\boldsymbol{\mu}}} &= \text{var}(\hat{\boldsymbol{\mu}}) = \sigma_w^2 (\mathbf{\Gamma}^T \mathbf{\Gamma})^{-1} \\ &= \sigma_w^2 \left( \begin{bmatrix} \mathbf{c}^T & \dots & \mathbf{c}^T \\ \vdots \\ \mathbf{c} \end{bmatrix} \right)^{-1} = \frac{1}{n} \sigma_w^2 (\mathbf{c}^T \mathbf{c})^{-1}. \end{aligned} \quad (\text{A2})$$

- (2) For random tooling deviations,  $\mathbf{v}^i \neq \mathbf{0}, \forall i$ . As the covariance of  $\mathbf{y}$  is not diagonal, the linear mixed model is used to estimate the mean shift and its covariance. First, the covariance of  $\mathbf{y}$  is determined by taking covariance on both sides of (4) and

the result is the following:

$$\begin{aligned}
 \Sigma_y = \text{var}(\epsilon) &= \text{var} \begin{pmatrix} \mathbf{Cv}^1 + \mathbf{w}^1 \\ \vdots \\ \mathbf{Cv}^n + \mathbf{w}^n \end{pmatrix} = \text{var} \begin{pmatrix} \mathbf{Cv}^1 \\ \vdots \\ \mathbf{Cv}^n \end{pmatrix} + \text{var} \begin{pmatrix} \mathbf{w}^1 \\ \vdots \\ \mathbf{w}^n \end{pmatrix} \\
 &= \text{var} \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{v}^1 \\ \vdots \\ \mathbf{v}^n \end{pmatrix} + \text{var} \begin{pmatrix} \mathbf{w}^1 \\ \vdots \\ \mathbf{w}^n \end{pmatrix} \\
 &= \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \Sigma_u & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \Sigma_u \end{bmatrix} \begin{bmatrix} \mathbf{C}^T & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{C}^T \end{bmatrix} + \sigma_w^2 \mathbf{I}_{(nm)} \\
 &= \begin{bmatrix} \mathbf{C}\Sigma_u\mathbf{C}^T & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{C}\Sigma_u\mathbf{C}^T \end{bmatrix} + \sigma_w^2 \mathbf{I}_{(nm)} = \begin{bmatrix} \mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I} \end{bmatrix}. \tag{A3}
 \end{aligned}$$

By applying the linear mixed model, the estimated  $\hat{\mu}$  is determined as follows:

$$\begin{aligned}
 \hat{\mu} &= (\Gamma^T \Sigma_y^{-1} \Gamma)^{-1} \Gamma^T \Sigma_y^{-1} \mathbf{y} = \begin{pmatrix} \begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C} \end{bmatrix}^T \begin{bmatrix} \mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C} \end{bmatrix} \\
 &\times \begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C} \end{bmatrix}^T \begin{bmatrix} \mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{y} \\ \vdots \\ \mathbf{y} \end{bmatrix} \\
 &= \begin{pmatrix} \begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C} \end{bmatrix}^T \begin{bmatrix} (\mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & (\mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C} \end{bmatrix} \\
 &= \left( \sum_{i=1}^n \mathbf{C}^T (\mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} \mathbf{C} \right)^{-1} \left( \sum_{i=1}^n \mathbf{C}^T (\mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} \mathbf{y}^i \right). \tag{A4}
 \end{pmatrix}
 \end{aligned}$$

The covariance of  $\hat{\mu}$  is determined as

$$\begin{aligned}
 \Sigma_{\hat{\mu}} &= \text{var}(\hat{\mu}) = (\Gamma^T \Sigma_y^{-1} \Gamma)^{-1} = \begin{pmatrix} \begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C} \end{bmatrix}^T \begin{bmatrix} \mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C} \end{bmatrix} \\
 &= \begin{pmatrix} \begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C} \end{bmatrix}^T \begin{bmatrix} (\mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} & \mathbf{0} \\ \vdots & \vdots \\ \mathbf{0} & (\mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{C} \\ \vdots \\ \mathbf{C} \end{bmatrix} \\
 &= \left( \sum_{i=1}^n \mathbf{C}^T (\mathbf{C}\Sigma_u\mathbf{C}^T + \sigma_w^2 \mathbf{I})^{-1} \mathbf{C} \right)^{-1}. \tag{A5}
 \end{pmatrix}
 \end{aligned}$$

## Appendix B: Tolerance allocation<sup>21</sup>

To find a function representing the ellipse  $f(\mathbf{u}, \boldsymbol{\sigma}, \xi)$ , suppose KPCs are the deviation distance from the nominal of each measurement point and are represented by  $y_i$  for  $i=1, \dots, m$ . The relationship between KPCs and KCCs is linear and expressed as  $\mathbf{C}\mathbf{u}=\mathbf{y}$ , where  $\mathbf{C}$  is a coordinate transformation from the fixture error vector to the part locating error and  $\mathbf{u}$  is a  $p$ -dimensional vector of tooling deviation. Let  $\{T_{y_i}\}_{i=1}^m$  be the allowable tolerances for the KPCs. Suppose that  $\boldsymbol{\sigma}=[\sigma_{p_1}, \dots, \sigma_{p_p}]^T$  is a vector of the allocated variances of the allowable tolerances for the KCCs. The tolerances for KCCs are allocated the following steps:

Step 1: For  $i=1, \dots, m$ , define  $\mathbf{z}_i \equiv \mathbf{c}_i / T_{y_i}$  where  $\mathbf{c}_i = [\mathbf{C}_{i1} \ \dots \ \mathbf{C}_{ip}]$ .

Step 2: For  $i=1, \dots, m$ , set  $\bar{\mathbf{z}}_i \equiv [z_{i,1}^2 \ \dots \ z_{i,p}^2]^T$ .

Step 3: Cost function:  $F(\boldsymbol{\sigma}) = \prod_{j=1}^p 1/\sigma_{p_j}$ .

Step 4: For given  $\xi$ ,  $K$  is the  $1-\xi$  percentile of the  $\chi_p^2(\xi)$ . Minimize  $F(\boldsymbol{\sigma})$  subject to  $\bar{\mathbf{y}}^T \boldsymbol{\sigma} \leq 1/K$  ( $i=1, \dots, 9$ ) and  $\sigma_{p_1}, \dots, \sigma_{p_p} > 0$ . The allocated tolerances of  $x_i$  is determined by the  $i$ th element of  $\boldsymbol{\sigma}$ . Therefore, the tolerance ellipsoidal region for the KCCs is determined as  $\sum_{j=1}^p (u_j^2 / \sigma_{p_j}^2) \leq K$ , i.e.  $f(\mathbf{u}, \boldsymbol{\sigma}, \xi)$  is represented by  $\sum_{j=1}^p u_j^2 / \sigma_{p_j}^2 = K$ .

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