

Optimal Process Adjustment by Integrating Production Data and Design of Experiments

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This paper proposes a method to improve the process model estimation based on limited experimental data by making use of abundant production data and to achieve the optimal process adjustment based on the improved process model. The proposed method is called an Estimation-adjustment (EA) method. Furthermore, this paper proves three properties associated with the EA, which guarantee the feasibility and effectiveness of using EA for integrating production and experimental data for optimal process adjustment. Also, the paper develops a sequential hypothesis testing procedure for implementing the EA. The properties and implementation of the EA are demonstrated in a cotton spinning process. Copyright © 2010 John Wiley & Sons, Ltd.

Keywords: process adjustment; design of experiments; sequential hypothesis testing

1. Introduction

Design of experiments (DOE) has been widely used in industrial processes to study the relationship between the input and output of a process, which provides a basis for adjusting the process input in order to achieve a target output. Abundant research exists in the DOE literature on how to design efficient experiments and how to perform effective modeling and analysis of the experimental data^{1–5}. Essentially, DOE is an offline technique, which usually happens at the design stage of the process and product (i.e. prior to production). In practice, the number of test samples that can be used in a designed experiment is usually limited due to timing, economical, or availability reasons. This leads to a small sample size of the experimental data, which further leads to large uncertainty in the process model that is estimated from the data. As the process model is a key in determining how to adjust the process input so as to achieve a target output, a process model with large uncertainty may lead to ineffective process adjustment.

With the rapid advancement in in-process sensing technologies, massive production data can now be continuously collected during the production⁶. In contrast with the experimental data, these production data are much easier to obtain, associated with less cost, and come with myriad amounts. Therefore, it is highly desirable to investigate how to fully utilize the production data to adaptively tune the initial process model estimated from the experimental data, as the production data become available. This has a potential benefit of reducing the possibly large uncertainty in the initial process model estimated and consequently making it possible to achieve the optimal process input adjustment.

The idea of integrating DOE and production data has been explored in the literature of engineering process control (EPC) and run to run control (R2R)^{7–13}. EPC and R2R are usually applied to dynamic processes, such as chemical and semiconductor processes, whose output has a tendency to shift or slowly drift away from the target due to uncontrollable process disturbance. The purpose is to adjust some controllable input variable(s) in order to compensate for the shift/drift and keep the process output on target. Moreover, EPC and R2R have been enhanced by combining with statistical process control (SPC)^{14–20}.

Compared with EPC and R2R, the research in this paper has the similarity of using DOE data to estimate an initial process model and then tuning the process model based on continuously available production data. The difference lies in the following aspects:

First, we assume that the true process model parameters do not change over time. As a result, our purpose of using production data for process model tuning is to reduce the large uncertainty in the estimation of the unknown process model parameters due to small experimental sample sizes. However, the purpose for process modeling tuning in EPC and R2R is to account for

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the process change (e.g. shifts or drifts) over time. For example, a typical R2R controller for a single-input single-output (SISO) process may assume the intercept of the linear process model to be time-varying; hence, the intercept needs to be updated recursively upon production data becoming available^{21, 22}. In fact, small experimental sample sizes have not been the focus of EPC and R2R. For example, in the SISO process discussed previously, the slope of the linear process model is assumed to be a constant and is estimated accurately using experimental data.

Second, EPC and R2R assume that it is the uncontrollable process disturbance that shifts/drifts the process output to be off target. Therefore, the effort of EPC and R2R is to adjust the process input to compensate for the shifts/drifts so as to keep the process output on target. However, we consider the process output to be off target because of inappropriate process input settings, which are caused by the large uncertainty in the process model estimation. Therefore, our effort in this research would be to reduce the uncertainty by incorporating the production data and identifying the optimal process input.

Specifically in this paper, we propose a method, called the Estimation-adjustment (EA) method, which uses production data to adaptively tune the initial DOE model which is likely to be imprecise due to limited sample sizes. To realize the proposed EA method, we prove three properties associated with the EA, which guarantee the feasibility and the effectiveness of using EA for integrating production data and DOE for optimal process adjustment. Briefly, the three properties of EA are: (i) there is a permanent offset between the estimates and the true values for the process model parameters due to insufficient perturbation in the process input; (ii) despite (i), i.e. the estimates for the process model parameters do not converge to the true model parameters as the EA procedure proceeds, it is still possible for the process output to converge to the target output; (iii) after the process output converges to the target, it will not drift away even when the EA procedure continues on. Based on these properties of EA, we further develop a statistical hypothesis testing procedure for implementing the EA.

The remainder of this paper is organized as follows: Section 2 is the problem formulation, in which we define the scope of the industrial processes under consideration and propose the EA method; Section 2 discusses the three properties of the EA and shows the detailed mathematical proof of each property; Section 3 proposes a sequential hypothesis testing procedure for implementing the EA; finally, Section 4 demonstrates the properties and implementation of the EA in a cotton spinning process.

2. Problem formulation

In this paper, we consider a process with an output variable, Y , and multiple input variables, X and \mathbf{Z} . X and \mathbf{Z} represent the controllable variable and uncontrollable variables, respectively. Furthermore, we consider that the relationship between the output and input variables can be represented by a multiple linear regression, called a 'process model', i.e.

$$Y = \alpha_0 + \alpha_X X + \alpha_Z^T \mathbf{Z} + \alpha_{XZ}^T \mathbf{Z} X + e_Y, \quad (1)$$

where e_Y is the residual error. A common assumption is that $e_Y \sim N(0, \sigma_{e_Y}^2)$ and e_Y is independent of X and \mathbf{Z} . Note that (1) is suitable for the following two types of processes: (i) the process with one controllable variable; (ii) the process with multiple controllable variables, but we can only choose one of them to adjust due to cost or engineering constraints. In the second type of process, in order for (1) to be an appropriate process model for the process, we need to change the definition of X and \mathbf{Z} as follows: we use X to denote the controllable variable that is chosen to be adjusted and use \mathbf{Z} to denote other controllable variables as well as uncontrollable variables.

Furthermore, we propose a simpler but equivalent representation for the process model in (1), for the convenience of the subsequent discussion in the paper. Specifically, let \mathbf{Z} be expressed by a sum of its mean vector, $\boldsymbol{\mu}_Z$, and a random vector, \mathbf{e}_Z , i.e.

$$\mathbf{Z} = \boldsymbol{\mu}_Z + \mathbf{e}_Z. \quad (2)$$

Inserting (2) into (1),

$$Y = (\alpha_0 + \alpha_Z^T \boldsymbol{\mu}_Z) + (\alpha_X + \alpha_{XZ}^T \boldsymbol{\mu}_Z) X + (\alpha_{XZ}^T \mathbf{e}_Z X + \alpha_Z^T \mathbf{e}_Z + e_Y).$$

Letting $\alpha_0 + \alpha_Z^T \boldsymbol{\mu}_Z \equiv \beta_0$, $\alpha_X + \alpha_{XZ}^T \boldsymbol{\mu}_Z \equiv \beta_1$ and $\alpha_{XZ}^T \mathbf{e}_Z X + \alpha_Z^T \mathbf{e}_Z + e_Y \equiv \varepsilon$, then the process model in (1) becomes

$$Y = \beta_0 + \beta_1 X + \varepsilon. \quad (3)$$

The goal of process adjustment is to find a setting for the controllable input variable, X , to make the output, Y , satisfy a predefined condition. In this paper, we focus on a setting for X that is optimal in the sense that it brings the mean output to the target. Specifically, let T denote the target value for Y . Then, the optimal setting for X should be $x_{opt} = (T - \beta_0) / \beta_1$, because $E(Y|X = x_{opt}) = E(\beta_0 + \beta_1 x_{opt} + \varepsilon) = T + E(\varepsilon) = T$. To find the optimal setting for X , we first must estimate the process model parameters, β_0 and β_1 , from data.

The data used to estimate the process model parameters may be collected from an offline designed experiment. Let $\mathbf{x}^T = [x'_1, \dots, x'_m]^T$ and $\mathbf{y}^T = [y'_1, \dots, y'_m]^T$ denote the data for X and Y , respectively. m is the sample size. If the commonly used orthogonal designs are used in the experiment, the data for X will have setting '-1' for the first $m/2$ samples and '1'

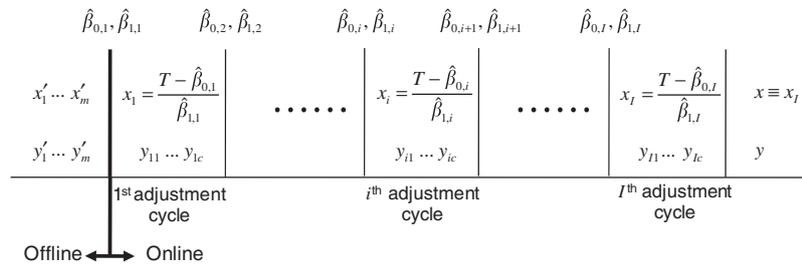


Figure 1. The proposed Estimation-adjustment (EA) method

for the rest of the samples, i.e. $\mathbf{x}^T = [x'_1, \dots, x'_{m/2}, x'_{m/2+1}, \dots, x'_m]^T = [-1, \dots, -1, 1, \dots, 1]^T$. These data can be used to estimate β_0 and β_1 .

However, the sample size of the experimental data is usually very limited, making the estimates for the process model parameters unreliable. On the other hand, after the production starts, massive amounts of production data can now be collected. This motivates the idea of taking advantage of the production data to update the initial, experimental data-based estimates for the process model parameters. Specifically, the proposed idea is as follows:

Denote the initial, experimental data-based estimates for β_0 and β_1 by $\hat{\beta}_{0,1}$ and $\hat{\beta}_{1,1}$, respectively. $\hat{\beta}_{0,1}$ and $\hat{\beta}_{1,1}$ are used to obtain the first setting for the process input, X , i.e. $x_1 = (T - \hat{\beta}_{0,1}) / \hat{\beta}_{1,1}$. Then, the offline experimental stage ends and the production starts. During the production, assume that the process adjustment cycle has length c , i.e. the process input will be adjusted after every c samples have been produced. This implies that the process input for the first c samples after the production starts will all be equal to x_1 . Denote the corresponding process output by $[y_{11}, \dots, y_{1c}]^T$. Upon the first c pairs of input-output data becoming available, they will be used, in conjunction with the offline experimental data, to update the estimates for β_0 and β_1 . Denote the updated estimates by $\hat{\beta}_{0,2}$ and $\hat{\beta}_{1,2}$, which will then be used to obtain the setting for the process input for the next process adjustment cycle, i.e. $x_2 = (T - \hat{\beta}_{0,2}) / \hat{\beta}_{1,2}$. This procedure will continue until the l th process adjustment cycle is finished (here, a stopping criterion is needed and will be defined later). Afterwards, the process input will be kept at x_l . Because this procedure involves progressively updating the estimates for β_0 and β_1 , and adjusting the process input accordingly, it is called an Estimation-adjustment (EA in short) method in this paper. The proposed EA method can be clearly depicted by Figure 1, in which $\hat{\beta}_{0,i}$ and $\hat{\beta}_{1,i}$ denote the estimates for β_0 and β_1 at the end of the $(i-1)$ th process adjustment cycle or equivalently at the start of the i th process adjustment cycle, based on the data collected in the offline experiment and in the first $(i-1)$ process adjustment cycles, $x_i = (T - \hat{\beta}_{0,i}) / \hat{\beta}_{1,i}$ denote the setting for the process input in the i th process adjustment cycle, and $[y_{i1}, \dots, y_{ic}]^T$ denote the data on the process output corresponding to x_i , $i = 1, \dots, l$.

There are two parameters associated with the EA method: the total number of adjustment cycles, l , and the length of each cycle, c . In Sections 3 and 4 of this paper, properties of the EA will be discussed, based on which methods for selecting the two parameters will be further developed.

Note that the EA involves estimating the process model parameters at the end of the offline experimental stage as well as at the end of each process adjustment cycle. To determine the estimation method, the property of the process model in (3) must be understood. A special property of the process model is that the data of the process output Y according to this process model are independently but non-identically distributed. More specifically, the data are independent, because $cov(y_i, y_j) = cov(\varepsilon_i, \varepsilon_j) = 0$ where y_i and y_j denote two samples for Y , and ε_i and ε_j are the corresponding residual errors. Recall that in the process model in (3), the residual error is $\varepsilon = \alpha_{XZ}^T \mathbf{e}_Z X + \alpha_Z^T \mathbf{e}_Z + e_Y$. The data are not identically distributed because the residual error ε is a function of the process input X . As a result, $Var(Y) = Var(\varepsilon) = X^2 var(\alpha_{XZ} \mathbf{e}_Z) + var(\alpha_Z \mathbf{e}_Z) + var(e_Y) + 2X cov(\alpha_{XZ} \mathbf{e}_Z, \alpha_Z \mathbf{e}_Z)$ i.e. the variance of Y is a function of the input variable X and thus it is not a constant across different samples. This special property of our process model, i.e. the samples being independently but non-identically distributed, is called 'heteroskedasticity' in statistics²³. To estimate the parameters of a model with heteroskedasticity, least-square estimation (LSE) can still be used because it has been proven to be unbiased and efficient given a large sample size²³. In our case, although the 'large sample size' condition may not be satisfied at the offline experimental stage and the first few process adjustment cycles, this condition will be satisfied as the process adjustment goes along. Essentially, the proposed EA method has the advantage of compensating for the sample shortage in the offline experiment by making use of the abundant data collected during the production. Therefore, in this paper, we propose to use LSE to estimate the process model parameters in the EA method.

3. Properties of the EA method

The purpose of the EA is to progressively adjust the process input X , until the optimal input is achieved. Recall that the optimal input is $x_{opt} = (T - \beta_0) / \beta_1$, which brings the mean output, $E(Y)$, to the target T . To achieve this purpose, it is desirable that the estimates for β_0 and β_1 , i.e. $\hat{\beta}_{0,i}$ and $\hat{\beta}_{1,i}$, should converge to β_0 and β_1 as the EA procedure goes along. However, the convergence is difficult to achieve, because the settings of the process input in the EA, i.e. the x_i 's, $i = 1, \dots, l$, do not have enough

perturbation, i.e. the x_i 's are too close to each other. This leads to a permanent offset between the estimates and the true values for β_0 and β_1 no matter how far along the EA goes—a unique property of the EA method, as summarized below:

Property I of EA: there is a permanent offset between the estimates and the true values for the process model parameters β_0 and β_1 .

Although this property shows that the estimates for β_0 and β_1 will not be able to converge to the true β_0 and β_1 due to a permanent offset in the estimates, Property II below shows that the mean process output may still reach the target—another unique property of the EA:

Property II of EA: it is possible for the mean process output to reach the target, at the existence of a permanent offset between the estimates and the true values for the process model parameters.

Finally, the third unique property of the EA method is stated below:

Property III of EA: once the mean process output reaches the target, it will not drift away even when the EA continues on.

Proofs of all the three properties can be found in the Appendix.

4. Implementation of EA by a sequential hypothesis testing method

Recall that there are two parameters associated with the EA: the total number of adjustment cycles, l , and the length of each cycle, c . To determine the total number of adjustment cycles is equivalent to defining a stopping criterion for the EA. The EA should stop once the mean process output reaches the target. We have known from Property II of EA that it is possible for the mean process output to reach the target although there is a permanent offset between the estimates and the true values for the process model parameters. This motivates the idea of using hypothesis testing to test the equality between the mean process output and the target in each process adjustment cycle. The hypothesis testing should be 'sequential' in the sense that if the mean process output is unequal to the target in a particular adjustment cycle, the testing should proceed to the next adjustment; otherwise, the testing will stop, i.e. the EA will stop. Specifically, at the i th adjustment cycle, the following hypotheses can be tested:

$$\begin{aligned} H_{0,i}: E(y_{ij}) &= T, \\ H_{1,i}: E(y_{ij}) &\neq T. \end{aligned} \quad (4)$$

If $H_{0,i}$ is rejected, then a similar test to that in (4) will be performed for the $(i+1)$ th adjustment cycle. Otherwise, the EA will stop, implying that the total number of adjustment cycle is $l=i$, and the process input will be set at x_i permanently to guarantee the mean process output to stay at the target.

To test the hypotheses in (4), a test statistic must be identified. A natural choice of the test statistic is

$$t_{0,i} = \frac{\bar{y}_{i\bullet} - T}{\sqrt{\text{var}(\bar{y}_{i\bullet})}} = \frac{\bar{y}_{i\bullet} - T}{\sqrt{\hat{\text{var}}(y_{ij})/c}}, \quad (5)$$

where $\bar{y}_{i\bullet} = \sum_{j=1}^c y_{ij}/c$. $t_{0,i}$ follows a student-t distribution with $c-1$ degrees of freedom, when $H_{0,i}$ is true. Therefore, $|t_{0,i}|$ can be compared with $t_{\alpha/2, c-1}$, i.e. the upper- $\alpha/2$ percentage point of the student-t distribution with $c-1$ degrees of freedom. If $|t_{0,i}| > t_{\alpha/2, c-1}$, then $H_{0,i}$ is rejected.

One risk associated with the sequential hypothesis testing method is Type-II error, β_{II} , which is the probability that the EA stops (i.e. $H_{0,i}$ is not rejected) when the mean process output has not reached the target. Let δ be the difference between the mean process output and the target T , i.e. $E(y_{ij}) = T + \delta$. Then, it can be derived that

$$\beta_{II} = F\left(t_{\alpha/2, c-1} - \frac{\delta}{\sqrt{\hat{\text{var}}(y_{ij})/c}}\right) - F\left(-t_{\alpha/2, c-1} - \frac{\delta}{\sqrt{\hat{\text{var}}(y_{ij})/c}}\right), \quad (6)$$

where $F(\bullet)$ is the cumulative probability function of the student-t distribution with $c-1$ degrees of freedom. The consequence of the Type-II error is severe and costly, because once it is committed, the process input will be permanently set to a value that creates a deviation between the mean process output and the target, i.e. the mean process output will never have a chance to reach the target.

The other risk associated with the sequential hypothesis testing method is Type-I error, which is the probability that the EA continues, although the mean of the process output has already reached the target. The consequence of Type-I error is usually not severe because (i) it is known from Property III of EA that after the mean process output reaches the target, it will not drift away even when the EA continues; (ii) even when we fail to identify that the mean process output reaches the target at a particular adjustment cycle due to a Type-I error, there is still a chance to make this identification at later adjustment cycles; and (iii) the cost of Type-I error, incurred by additional unnecessary process adjustment cycles, is relatively low, compared with the cost of Type-II error.

Considering that the consequence of Type-II error is severe and costly, Type-II error needs to be well controlled. Therefore, in what follows, we will show how to modify the sequential hypothesis testing method presented previously in order to guarantee that Type-II error will not exceed a pre-specified upper bound, $\beta_{II,U}$:

It can be proved that Type-II error is monotonically decreasing with respect to the length of the adjustment cycle, c . Therefore, if the Type-II error of the hypothesis testing for a particular adjustment cycle is larger than the pre-specified upper bound, i.e. $\beta_{II} > \beta_{II,U}$, we can lower the Type-II error by collecting additional samples, i.e. by increasing the length of the adjustment cycle.

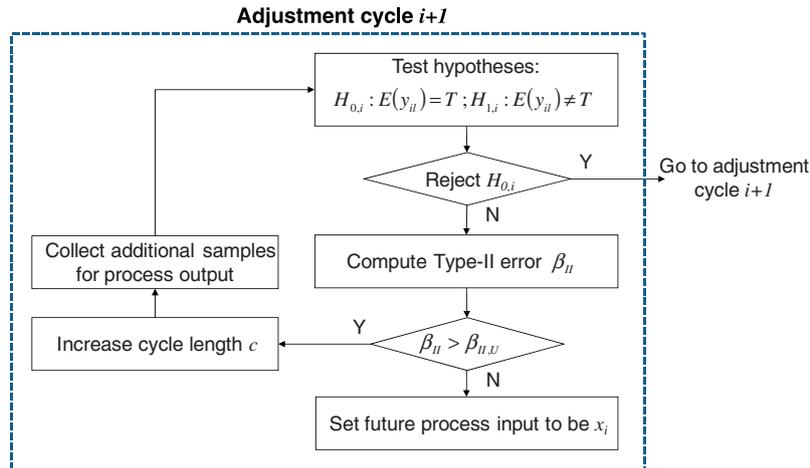


Figure 2. A sequential hypothesis testing method for implementing the EA considering an upper bound for Type-II error

On the other hand, increasing the length of an adjustment cycle changes the hypothesis testing for this adjustment cycle by changing the test statistic $t_{0,j}$ in (5) and the test threshold $t_{\alpha/2, m+ic-2}$; hence, the testing must be re-conducted. While the original testing (before c is increased) concludes that $H_{0,j}$ cannot be rejected, the re-conducted testing may make a different conclusion that $H_{0,j}$ is rejected. As a result, the EA will continue.

When the EA stops again, β_{II} will be re-computed and compared with $\beta_{II,U}$. If $\beta_{II} > \beta_{II,U}$, additional samples are collected at the corresponding adjustment cycle and the hypothesis testing at that cycle is re-conducted. This process repeats until $\beta_{II} \leq \beta_{II,U}$.

As a summary, Figure 2 shows a flow chart of the revised sequential testing method.

5. Examples

The EA method will be demonstrated in a cotton spinning process²⁴, in which the process output variable is skein strength. Because skeins are made of fibers, the input variables of this process are fiber fineness and fiber strength. Fiber fineness is considered to be controllable, because producers have the option to choose fibers with different grades of fineness. However, even fibers with the same grade of fineness may have natural variation in the fiber strength; hence, fiber strength is considered to be uncontrollable. Furthermore, each input and output variable is standardized by subtracting its mean and then being divided by its standard deviation. The resulting standardized variables are denoted by Y (standardized skein strength), X (standardized fiber fineness), and Z (standardized fiber strength), all being assumed to follow the standard normal distribution.

The output is linked to the input variables by the following regression (the true process model):

$$Y = -0.343X + 0.602Z + 0.582XZ + e_Y. \quad (7)$$

Because Z has been standardized, $Z = e_Z$. Then, (7) becomes $Y = -0.343X + 0.602e_Z + 0.582Xe_Z + e_Y$. Letting $\beta_0 = 0$, $\beta_1 = -0.343$, and $\varepsilon = 0.602e_Z + 0.582Xe_Z + e_Y$, then (7) can be further written as:

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

In other words, the true process model parameters are $\beta_0 = 0$ and $\beta_1 = -0.343$.

The target skein strength is its mean, indicating that the target value for standardized skein strength, Y , is zero, i.e. $T = 0$. Thus, the optimal setting for the standardized fiber length, X , should be $x_{opt} = (T - \beta_0) / \beta_1 = 0$.

Based on the above understanding of the cotton spinning process, two simulation studies will be conducted: one is to demonstrate the properties of the EA procedure, and the other is to show how to implement the EA procedure by the sequential hypothesis testing method developed in Section 4.

5.1. Demonstration of EA properties in cotton spinning process

To demonstrate Property I, the following simulation steps are developed. These steps will be performed for given values for m (sample size of the offline experiment), c (length of each adjustment cycle), and l (total number of adjustment cycles).

S1 (generate offline experimental data): Set X to be -1 for the first $m/2$ samples and 1 for the next $m/2$ samples. Apply (7) to generate Y for each sample. Then, use LSE to obtain initial estimates for β_0 and β_1 , i.e. $\hat{\beta}_{0,1}$ and $\hat{\beta}_{1,1}$, respectively.

S2 (generate online data and conduct the EA procedure): Let $i = 1$

S2.1: Set X to be $x_i = (T - \hat{\beta}_{0,i}) / \hat{\beta}_{1,i}$ in the i th adjustment cycle.

S2.2: With $X = x_i$, apply (7) to generate Y for c samples in the i th adjustment cycle.

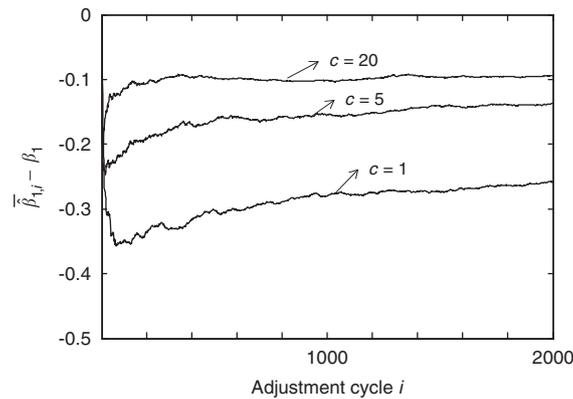


Figure 3. Offset between the estimate and true value for β_1 as the EA proceeds

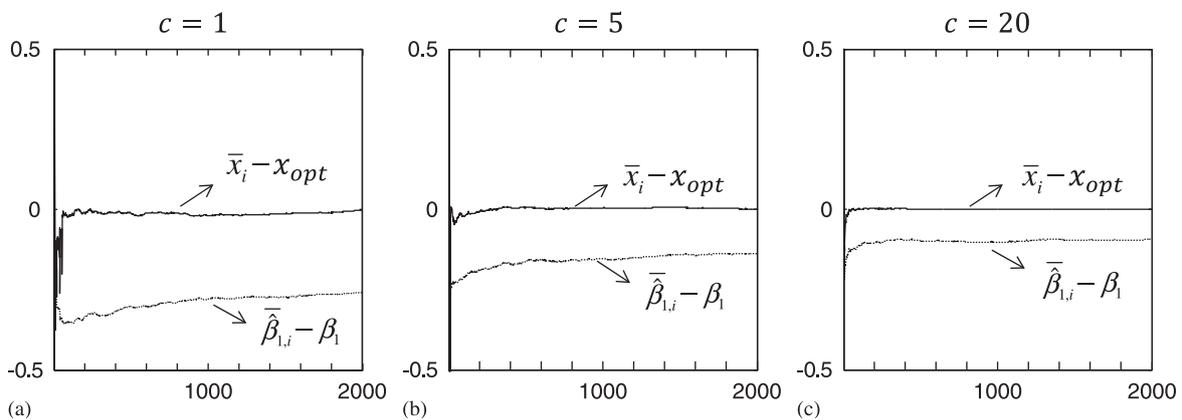


Figure 4. Difference between the process input at each adjustment cycle, \bar{x}_i , and the optimal input, \bar{x}_{opt} , as the EA procedure proceeds

S2.3: Use all data generated so far (including the offline experimental data) to re-estimate β_0 and β_1 , i.e. to obtain $\hat{\beta}_{0,i+1}$ and $\hat{\beta}_{1,i+1}$.

S2.4: Let $i=i+1$. If $i \leq l$, go to S2.1; otherwise, exit.

By executing the above simulation steps, we can obtain a sequence consisting of progressively updated estimates for the true process model parameters $\beta = [\beta_0, \beta_1]^T$ after each adjustment cycle. Denote this sequence by $\hat{\beta}_2, \dots, \hat{\beta}_{l+1}$. We can execute the simulation for N times and compute the average sequence, $\bar{\beta}_2, \dots, \bar{\beta}_{l+1}$. Then, we can compare the average sequence with the true process model parameters.

For example, consider that four samples are generated offline, i.e. $m=4$. Figure 3 shows the offset between $\bar{\beta}_{1,i}$ and the true model parameter $\beta_1 = -0.343$, for different lengths of the adjustment cycle, i.e. $c=1, 5, 20$. It can be seen that $\bar{\beta}_{1,i}$ does not converge to β_1 even when the adjustment cycle goes as large as 2000. Furthermore, comparison across the three curves in Figure 3 shows that the larger the length of the adjustment cycle, c , the smaller the offset between $\bar{\beta}_{1,i}$ and β_1 . Similar phenomena can be observed when comparing $\bar{\beta}_{0,i}$ and the true model parameter $\beta_0 = 0$.

Furthermore, to demonstrate Property II of EA, the mean process output in each adjustment cycle, $E(y_{ij})$, needs to be compared with the target, T , $i=1, \dots, l, j=1, \dots, c$. Because $E(y_{ij}) = \beta_0 + \beta_1 E(x_i)$, the comparison between $E(y_{ij})$ and T can be obtained by comparing $E(x_i)$ and $x_{opt} = (T - \beta_0) / \beta_1 = 0$. An estimate for $E(x_i)$ can be obtained by running simulation steps S1–S2 for N times and computing the average x_i , \bar{x}_i . For example, Figure 4 shows the difference between \bar{x}_i and $x_{opt} = 0$, for different lengths of the adjustment cycle, i.e. $c=1, 5, 20$. To make a contrast, the curves in Figure 3 are added into Figure 4. It can be clearly seen that while $\bar{\beta}_{1,i}$ does not converge to β_1 , \bar{x}_i converges to x_{opt} , i.e. the mean process output will converge to the target in the EA procedure.

In addition, Figure 4 also demonstrates Property III of EA. Specifically, it can be seen that \bar{x}_i converges to x_{opt} far before the 2000th adjustment cycle. After that, although the process adjustment is still continued, \bar{x}_i does not drift away from x_{opt} . In other words, the mean process output does not drift away from its target after the target has been reached.

5.2. Implementation of EA in cotton spinning process

Before the EA is implemented in the cotton spinning process, the following information is given:

- Offline experimental sample size is $m=4$;
- Online sample batch size is 5 (i.e. the length of each adjustment cycle can only be multiples of five);
- Type-I error of the hypothesis testing in each adjustment cycle is $\alpha=0.1$;
- The upper bound of Type-II error, when the mean process output deviates from the target by at least one standard deviation (i.e. $|\delta| \geq 1$), is $\beta_{II,U}=0.01$.

Based on the given information, the EA procedure is implemented. Detailed steps and results (including intermediate results for verification of the procedure) are shown as follows:

At the offline experimental stage:

- Initial estimates for β_0 and β_1 based on the offline experimental samples are $\hat{\beta}_{0,1}=0.7368$ and $\hat{\beta}_{1,1}=-0.0154$, respectively.
- The process input is set to be $x_1=(T-\hat{\beta}_{0,1})/\hat{\beta}_{1,1}=47.7338$.

At the first adjustment cycle ($i=1$):

- Five samples of the process output corresponding to the process input $x_1=47.7338$ are collected (i.e. $c=5$), based on which the hypothesis testing rejects $H_{0,1}$.

At the second adjustment cycle ($i=2$):

- Updated estimates for β_0 and β_1 based on all samples collected so far are $\hat{\beta}_{0,2}=0.7256$ and $\hat{\beta}_{1,2}=-0.5501$, respectively.
- The process input is set to be $x_2=(T-\hat{\beta}_{0,2})/\hat{\beta}_{1,2}=1.3192$.
- Five samples of the process output corresponding to the process input $x_1=1.3192$ are collected (i.e. $c=5$), based on which the hypothesis testing fails to reject $H_{0,2}$. Therefore, Type-II error is computed, $\beta_{II}=0.7364$.
- Because $\beta_{II}=0.7364>0.01$, five more samples of the process output are collected at this adjustment cycle (i.e. $c=10$), based on which the hypothesis testing still cannot reject $H_{0,2}$. Therefore, Type-II error is re-computed, $\beta_{II}=0.5157$.
- Because $\beta_{II}=0.5157>0.01$, five more samples of the process output are collected (i.e. $c=15$), based on which the hypothesis testing still cannot reject $H_{0,2}$. Therefore, Type-II error is re-computed, $\beta_{II}=0.1689$.
- Because $\beta_{II}=0.5157>0.01$, five more samples of the process output are collected (i.e. $c=20$), based on which the hypothesis testing rejects $H_{0,2}$.

At the third adjustment cycle ($i=3$):

- Updated estimates for β_0 and β_1 based on all samples collected so far are $\hat{\beta}_{0,3}=0.1036$ and $\hat{\beta}_{1,3}=-0.5373$, respectively.
- The process input is set to be $x_3=(T-\hat{\beta}_{0,3})/\hat{\beta}_{1,3}=0.1929$.
- Twenty samples of the process output are collected (i.e. $c=20$), based on which the hypothesis testing fails to reject $H_{0,3}$. Therefore, Type-II error is computed, $\beta_{II}=0.0140$.
- Because $\beta_{II}=0.0140>0.01$, five more samples of the process output are collected (i.e. $c=25$), based on which the hypothesis testing fails to reject $H_{0,3}$. Therefore, Type-II error is computed, $\beta_{II}=0.0012$.
- Because $\beta_{II}=0.0012<0.01$, the EA procedure stops.

In summary, the EA method went through three adjustment cycles with 50 samples collected online, before it stopped. The process input was finally set to be $X=x_3=0.1929$, resulting in a mean process output $E(Y)=\beta_0+\beta_1X=-0.0662$.

6. Conclusion

Process models estimated from limited experimental data may have large uncertainty, which leads to ineffective process adjustment. On the other hand, the wide adoption of automatic sensing technologies in modern industrial processes generates abundant production data. This motivates the use of the production data to progressively tune the imprecise process models estimated from the limited experimental data, and consequently enable optimal process adjustment.

Following this line of thinking, this paper proposes an EA method to improve the process model estimation based on limited experimental data by making use of abundant production data and to achieve the optimal process adjustment based on the improved process model. To realize the proposed EA method, we prove three unique properties associated with the EA, which guarantee the feasibility and effectiveness of using EA for integrating production data and DOE for optimal process adjustment. Based on these properties of EA, we further develop a sequential hypothesis testing procedure for implementing the EA. The uniqueness of the procedure is that it can take an upper bound for Type-II error into account, so that the Type-II error of the decision making can be well controlled. Finally, we demonstrate the properties and implementation of the EA in a cotton spinning process.

Future research directions may include extension of this research to processes with multiple outputs or/and multiple input variables to be controlled.

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Appendix A

A.1. Proof on Property I of EA

According to the EA method depicted in Section 2, $\hat{\beta}_i = [\hat{\beta}_{0,i}, \hat{\beta}_{1,i}]^T$ is obtained using LSE based on the data collected in the offline experiment as well as the data collected from the first through the $(i-1)$ th process adjustment cycle, i.e.

$$\hat{\beta}_i = (\mathbf{X}_{i-1}^T \mathbf{X}_{i-1})^{-1} \mathbf{X}_{i-1}^T \mathbf{Y}_{i-1} = (\mathbf{X}_{i-1}^T \mathbf{X}_{i-1})^{-1} \mathbf{X}_{i-1}^T (\mathbf{X}_{i-1} \beta + \varepsilon_{i-1}) = \beta + (\mathbf{X}_{i-1}^T \mathbf{X}_{i-1})^{-1} \mathbf{X}_{i-1}^T \varepsilon_{i-1}, \quad (\text{A1})$$

where $\beta = [\beta_0, \beta_1]^T$,

$$\mathbf{X}_{i-1} = \begin{bmatrix} 1 & \dots & 1 \\ -1 \dots -1 & 1 \dots 1 & x_1 \dots x_1 & \dots & x_{i-1} \dots x_{i-1} \end{bmatrix}^T,$$

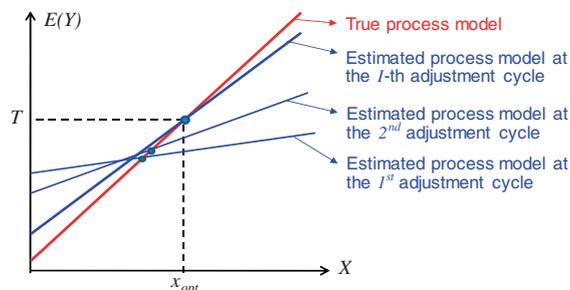


Figure A1. The mean process output achieves the target at the i th adjustment cycle although the estimated process model parameters have an offset with respect to the true process model parameters

$Y_{i-1} = [y'_{11}, \dots, y'_{m1}, y_{11}, \dots, y_{1c}, \dots, y_{i-1,1}, \dots, y_{i-1,c}]^T$, and $\varepsilon_{i-1} = [\varepsilon'_{11}, \dots, \varepsilon'_{m1}, \varepsilon_{11}, \dots, \varepsilon_{1c}, \dots, \varepsilon_{i-1,1}, \dots, \varepsilon_{i-1,c}]^T$. After performing matrix multiplication and inversion, (A1) becomes

$$\hat{\beta}_i = \beta + \frac{\begin{bmatrix} m+c \sum_{k=1}^{i-1} x_k^2 & -c \sum_{k=1}^{i-1} x_k \\ -c \sum_{k=1}^{i-1} x_k & m+c(i-1) \end{bmatrix} \begin{bmatrix} \sum_{h=1}^m \varepsilon'_h + \sum_{k=1}^{i-1} \sum_{l=1}^c \varepsilon_{kl} \\ -\sum_{h=1}^{m/2} \varepsilon'_h + \sum_{h=m/2+1}^m \varepsilon'_h + \sum_{k=1}^{i-1} \sum_{l=1}^c x_k \varepsilon_{kl} \end{bmatrix}}{m(m+c(i-1)+c \sum_{k=1}^{i-1} x_k^2) + c^2(i-1) \sum_{k=1}^{i-1} x_k^2 - c^2 (\sum_{k=1}^{i-1} x_k)^2} \quad (A2)$$

Divide both the numerator and denominator of the second term in the right-hand side of (A2) by $c(i-1)$. Then, when $c(i-1)$ goes large as the EA proceeds, it is evident that $m/(c(i-1)) \approx 0$ because the offline sample size m is usually very small; $\sum_{k=1}^{i-1} \sum_{l=1}^c \varepsilon_{kl} / (c(i-1)) \approx 0$ because $E(\varepsilon_{kl}) = E(\alpha_{XZ}^T \mathbf{e}_{Z_k} x_k + \alpha_Z^T \mathbf{e}_{Z_{kl}} + e_{Y_{kl}}) = 0$; and $\sum_{k=0=1}^{i-1} \sum_{l=1}^c x_k \varepsilon_{kl} / (c(i-1)) \approx 0$ because $E(x_k \varepsilon_{kl}) = 0$. Thus, (A2) becomes

$$\hat{\beta}_i \approx \beta + \frac{\begin{bmatrix} \hat{\sigma}_{x,i-1}^2 + (\hat{\mu}_{x,i-1})^2 & -\hat{\mu}_{x,i-1} \\ -\hat{\mu}_{x,i-1} & 1 \end{bmatrix} \begin{bmatrix} \sum_{h=1}^m \varepsilon'_h \\ -\sum_{h=1}^{m/2} \varepsilon'_h + \sum_{h=m/2+1}^m \varepsilon'_h \end{bmatrix}}{m(1 + \hat{\sigma}_{x,i-1}^2 + (\hat{\mu}_{x,i-1})^2) + \hat{\sigma}_{x,i-1}^2 c(i-1)}, \quad (A3)$$

where $\hat{\mu}_{x,i-1} = \sum_{k=1}^{i-1} x_k / (i-1)$ and $\hat{\sigma}_{x,i-1}^2 = \sum_{k=1}^{i-1} x_k^2 / (i-1) - (\hat{\mu}_{x,i-1})^2$.

When the settings for the process input, i.e. the x_k 's, $k=1, \dots, i-1$, are very close to each other (no enough perturbation), the sample variance of the x_k 's, i.e. $\hat{\sigma}_{x,i-1}^2$ will be very small. Therefore, $\hat{\sigma}_{x,i-1}^2 c(i-1)$ may still be small when $c(i-1)$ goes large. As a result, the denominator of the fraction in (A3) may not be big enough to make the whole fraction close to zero. This creates an offset in $\hat{\beta}_i$ with respect to β . Consider a special case when the x_k 's are exactly the same as each other, i.e. $x_k = x_1$. Then, $\hat{\sigma}_{x,i-1}^2 = 0$ and (A3) becomes

$$\hat{\beta}_i \approx \beta + \frac{\begin{bmatrix} x_1^2 & -x_1 \\ -x_1 & 1 \end{bmatrix} \begin{bmatrix} \sum_{h=1}^m \varepsilon'_h \\ -\sum_{h=1}^{m/2} \varepsilon'_h + \sum_{h=m/2+1}^m \varepsilon'_h \end{bmatrix}}{m(1+x_1^2)} = \beta + \frac{-x_1 \sum_{h=1}^m \varepsilon'_h - \sum_{h=1}^{m/2} \varepsilon'_h + \sum_{h=m/2+1}^m \varepsilon'_h}{m(1+x_1^2)} \begin{bmatrix} -x_1 \\ 1 \end{bmatrix},$$

in which the offset in $\hat{\beta}_i$ with respect to β under a small offline sample size m is obvious.

A.2. Proof on Property II of EA

Assume that the mean process output reaches the target at the l th adjustment cycle, i.e. $E(y_{ll}) = T, l=1, \dots, c$, and $x_l = x_{opt}$. Because $x_l = (T - \hat{\beta}_{0,l}) / \hat{\beta}_{1,l}$ and $x_{opt} = (T - \beta_0) / \beta_1$, $x_l = x_{opt}$ implies that

$$\frac{T - \hat{\beta}_{0,l}}{\hat{\beta}_{1,l}} = \frac{T - \beta_0}{\beta_1} \quad (A4)$$

According to Property I, there is an offset in the estimates for β_0 and β_1 ; hence, $\hat{\beta}_{1,l} \neq \beta_1$ and $\hat{\beta}_{0,l} \neq \beta_0$. Then, (A4) can be written as

$$\frac{\beta_0 - \hat{\beta}_{0,l}}{\hat{\beta}_{1,l} - \beta_1} = \frac{T - \beta_0}{\beta_1} \quad (A5)$$

i.e. $(\beta_0 - \hat{\beta}_{0,l}) / (\hat{\beta}_{1,l} - \beta_1) = x_{opt}$. Note that $(\beta_0 - \hat{\beta}_{0,l}) / (\hat{\beta}_{1,l} - \beta_1)$ is actually the solution for X in the equation $\beta_0 + \beta_1 X = \hat{\beta}_{0,l} + \hat{\beta}_{1,l} X$. In other words, it is the X -coordinate of the intersecting point between two lines, $E(Y) = \beta_0 + \beta_1 X$ (the true process model) and $E(Y) = \hat{\beta}_{0,l} + \hat{\beta}_{1,l} X$ (the estimated process model at the end of the $(l-1)$ th process adjustment cycle or equivalently at the beginning of the l th process adjustment cycle). An important indication of this is as follows: during the EA procedure, the line of the estimated process model will change as the process adjustment goes along. Consequently, the intersecting point between the line of the estimated process model and the line of the true process model will change. At the moment that the X -coordinate of this intersecting point becomes x_{opt} , the mean process output reaches the target, although the estimates for β_0 and β_1 at this moment may still have an offset with respect to the true values for β_0 and β_1 . Please see Figure A1 for a graphical illustration of this property.

A.3. Proof on Property III of EA

Assume that the mean process output reaches the target at the l th adjustment cycle. If the EA procedure continues on, then the estimates for β_0 and β_1 at the $(l+1)$ th adjustment cycle are:

$$\begin{aligned} \begin{bmatrix} \hat{\beta}_{0,l+1} \\ \hat{\beta}_{1,l+1} \end{bmatrix} &= \begin{bmatrix} \hat{\beta}_{0,l} \\ \hat{\beta}_{1,l} \end{bmatrix} + \frac{(\mathbf{x}_{l-1}^T \mathbf{x}_{l-1})^{-1} \begin{pmatrix} 1 \\ x_l \end{pmatrix} \left(\sum_{l=1}^c y_{ll} / c - [1 \ x_l] \begin{bmatrix} \hat{\beta}_{0,l} \\ \hat{\beta}_{1,l} \end{bmatrix} \right)}{1 + c[1 \ x_l](\mathbf{x}_{l-1}^T \mathbf{x}_{l-1})^{-1} \begin{pmatrix} 1 \\ x_l \end{pmatrix}} \\ &= \begin{bmatrix} \hat{\beta}_{0,l} \\ \hat{\beta}_{1,l} \end{bmatrix} + \frac{(\mathbf{x}_{l-1}^T \mathbf{x}_{l-1})^{-1} \begin{pmatrix} 1 \\ x_l \end{pmatrix} (\sum_{l=1}^c (\beta_0 + \beta_1 x_l + \varepsilon_{ll}) / c - (\hat{\beta}_{0,l} + \hat{\beta}_{1,l} x_l))}{1 + c[1 \ x_l](\mathbf{x}_{l-1}^T \mathbf{x}_{l-1})^{-1} \begin{pmatrix} 1 \\ x_l \end{pmatrix}}. \end{aligned} \tag{A6}$$

Because the mean process output has reached the target at the l th adjustment cycle, $x_l = x_{opt}$ and $\beta_0 + \beta_1 x_l = \hat{\beta}_{0,l} + \hat{\beta}_{1,l} x_l = T$. Therefore, (A6) can be further written as

$$\begin{bmatrix} \hat{\beta}_{0,l+1} \\ \hat{\beta}_{1,l+1} \end{bmatrix} = \begin{bmatrix} \hat{\beta}_{0,l} \\ \hat{\beta}_{1,l} \end{bmatrix} + \frac{(\mathbf{x}_{l-1}^T \mathbf{x}_{l-1})^{-1} \begin{pmatrix} 1 \\ x_l \end{pmatrix}}{1 + c[1 \ x_l](\mathbf{x}_{l-1}^T \mathbf{x}_{l-1})^{-1} \begin{pmatrix} 1 \\ x_l \end{pmatrix}} \times \frac{\sum_{l=1}^c \varepsilon_{ll}}{c}.$$

It is easy to see that the conditional mean of $[\hat{\beta}_{0,l+1}, \hat{\beta}_{1,l+1}]^T$ given $[\hat{\beta}_{0,l}, \hat{\beta}_{1,l}]^T$ is $[\hat{\beta}_{0,l}, \hat{\beta}_{1,l}]^T$, i.e. $E([\hat{\beta}_{0,l+1}, \hat{\beta}_{1,l+1}]^T | [\hat{\beta}_{0,l}, \hat{\beta}_{1,l}]^T) = [\hat{\beta}_{0,l}, \hat{\beta}_{1,l}]^T$. As a result, the mean process output at the $(l+1)$ th adjustment cycle, i.e. $y_{l+1,l}$, $l = 1, \dots, c$, will have a mean equal to $E(y_{l+1,l}) = \beta_0 + \beta_1 E(x_{l+1}) = \beta_0 + \beta_1 E((T - \hat{\beta}_{0,l+1}) / \hat{\beta}_{1,l+1}) = \beta_0 + \beta_1 E((T - \hat{\beta}_{0,l}) / \hat{\beta}_{1,l}) = T$.

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