Complexity metrics for mixed model manufacturing systems based on information entropy

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Abstract: Mixed model manufacturing systems are increasingly used to meet global competition by providing a broad variety of products to customers. The increase of product variety adds more complexity to production processes, thus, leading to a negative effect on the performance of production processes. Therefore, it is of great interest to effectively measure such complexity and to quantify its effect on manufacturing system performance. In this paper, a set of complexity metrics are proposed for measuring the complexity of different elements in a manufacturing system. These metrics were defined by constructing a linkage with the communication system's framework. Different from those existing complexity measures defined in the literature, this paper considers production quality into the measure of the process capability on handling the complexity induced by the input demand variety. Examples are given in the paper to discuss different properties of the defined metrics and their potential applications.

Keywords: complexity metrics; manufacturing systems; information theory; information entropy; mutual information; product variety; communication systems; quality; coefficient constraint; mixed model assembly lines; MMAL.

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1 Introduction

The increasing global competition demands manufacturing industry to move from mass production into mass customisation production in order to provide more product varieties to satisfy customer demands. For example, mixed model assembly lines (MMALs) are increasingly used in many assembly plants, where different assemble operations are performed to produce different type of products in the same production line. Although MMALs can provide better flexibility to meet customer demands with short production lead time, it brings out high requirements on handling the increased complexity of production operations. Therefore, at the design of a MMAL, it is essential not only to know how to measure the operation complexity, but also to assess how well a manufacturing process can handle such an operational complexity to meet the variety of customer demands.

Recently, some theoretical measures of process complexity have been proposed for manufacturing systems under different contexts. For example, Nakazawa and Suh (1984) and Suh (1990) measured the process complexity in terms of the amount of efforts needed to complete a product, where the effort was quantified based on the product requirement on geometric precision and surface quality. From an assembly operation point of view, Fujimoto and Ahmed (2001) used the concept of entropy by measuring the amount of the uncertainty in key assembly operations, such as gripping, positioning and inserting parts. From a product point of view, Fujimoto et al. (2003) defined a measure of complexity based on the product structure and also showed that the impact of product variety could be reduced if the complexity of the product structure was reduced. Moreover, Deshmukh et al. (1992) and Zhu et al. (2008) defined complexity to consider the demand variety based on the part mix ratio of different products.

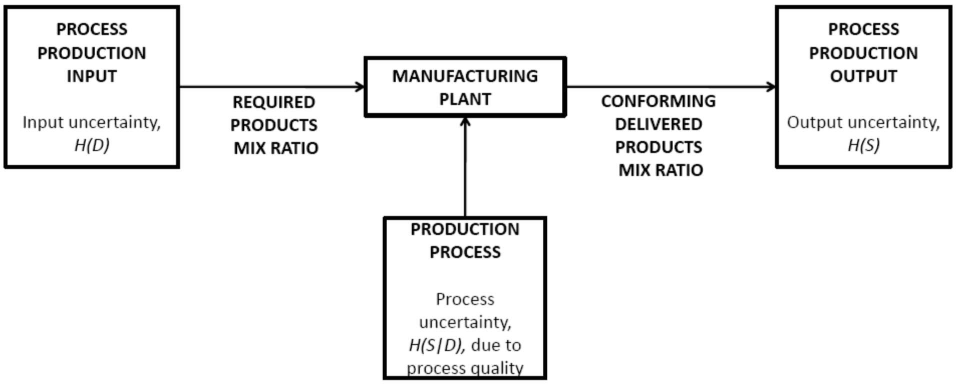
It has been shown that the increase of product variety has a negative impact on the manufacturing system performance (Fisher et al., 1995; MacDuffie et al., 1996; Niimi and Matsudaira, 1997). Deshmukh (1993) described the complexity of manufacturing systems by two aspects: the complexity induced by the demanded product varieties and the complexity induced to the plant when the number of operations and/or stations is increased. Recently, some new entropic-related complexity measures have been proposed to describe the uncertainty induced by the rescheduling of the production (Huaccho Huatuco et al., 2009) or the decision-making complexity for manufacturing organisations (Calinescu et al., 2001). Frizelle and Suhov (2008), based on three case studies, also proposed some forms of quantification of the system's innate complexity while considering data uncertainty due to measurement noises. All those previous studies mainly focused on how to describe the complexity of product demands or the manufacturing system itself. Differently, this paper focuses on how to describe the capability of a manufacturing process to handle the product demand complexity.

The objective of this paper is to further define a new quantitative metric, which can assess how well a manufacturing system can handle the complexity induced by the demanded input part mix ratio. The basic principle for defining such a metric is to assess whether the process can produce an output product mix ratio that meets the variety of the input part mix ratio as closely as possible. In practice, the output product mix ratio of a production process will not exactly match the input part mix ratio due to inevitable defective scraps produced by the production process. For example, Process A is designed to meet the demand consisting of two different product types with an input part mix ratio of 40% and 60% for type 1 and type 2, respectively. Assume that the production conforming quality is 98% for type 1 and 97% for type 2. In this case, the output product mix ratio becomes 39.2% for type 1 and 58.2% for type 2. Note that there are 2.6% scraps in the output product mix ratio. Based on the method in Zhu et al. (2008), we can compute the input complexity as 0.9710. In this case, the proposed metric for describing the process capability for handling complexity, which is called the normalised process capability for complexity (NPCC) to be described in Section 3, can be calculated as 97.62%. It should be notified that such an NPCC metric is related to production quality, but cannot be fully represented by production quality alone. For example, Process B is designed to produce a single type of products at a production conforming quality of 99%. Even though production quality of Process B is higher than that of Process A, the capability of Process B for handling demand complexity is not necessarily higher than that of Process A. In fact, in this special case, the amount of complexity handled by Process B is equal to zero because Process B has no demand input complexity. Therefore, the process capability metric for handling complexity should be defined to consider both factors of input mix ratio and process operation quality.

As shown in Figure 1, the input demand complexity of a manufacturing system is induced by the demanded product variety, which can be represented by the demand entropy $H(D)$ based on the demanded input mix ratio as defined in the existing literature (Zhu et al., 2008). If the process can ideally produce the output products to exactly match the input demand variety, the entropy of output uncertainty $H(S)$ should be the same as the input entropy $H(D)$. However, due to the inevitable uncertainty of process operation quality, the output products may not exactly match the input mix ratio. Therefore, the defined metrics should be able to describe the degree of matching between output entropy $H(S)$ and input entropy $H(D)$, which inspired us to employ the performance metrics used in communication channels.

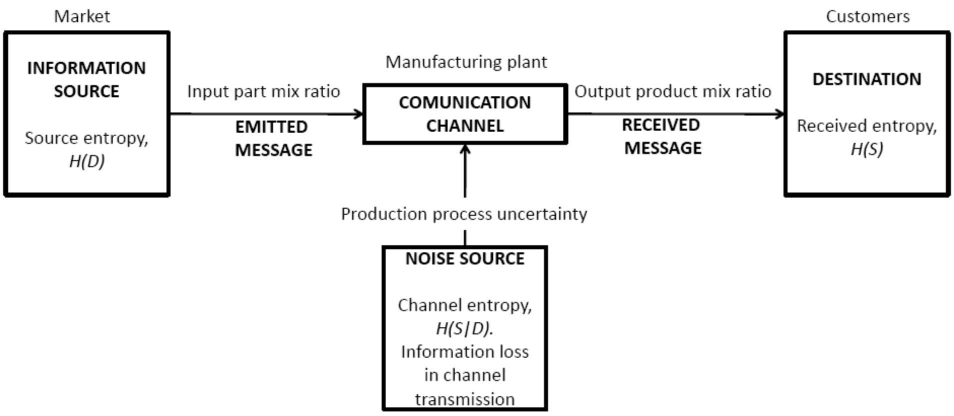
As it is known, the mutual information index $I(D, S)$ is defined to describe the dependency between random input codes D and output codes S for communication channel performance analysis (Shannon, 1948). In this paper, we will discuss in detail how this index can be used to assess the performance of a manufacturing process on handling the demand complexity. In this way, our work is an extension of the ideas in Zhu et al. (2008) by considering the effect of process quality in the performance measures of a production process on handling of the demand variety. In addition, all three elements of the complexity in a manufacturing system, i.e., input demand complexity, output product complexity and process capability for handling input complexity, are defined under a new integrated framework based on the principles used to analyse a communication system. Therefore, the well-developed communication channel analysis methods can be further applied in the future for optimising the manufacturing process capability in terms of handling demand complexity.

Figure 1 Manufacturing system representation



The rest of the paper is organised as follows. Section 2 presents the modelling of a manufacturing system in terms of the input/output relationship based on a communication system framework. Section 3 introduces the proposed metrics for measuring the performance of a manufacturing system, which are defined based on the information theory for analysing communication channels. In Section 4, two examples are used to illustrate different aspects in the use of the proposed metrics. A detail analysis on measuring complexity for a two-station production process is discussed in Section 5. Furthermore, Section 6 is used to present some potential uses of our proposed metrics. Finally, Section 7 concludes the paper.

Figure 2 Mapping of elements from a communication system into a manufacturing system



2 Modelling manufacturing systems by using a communication systems framework

This section is at the first time to show the linkage of the input/output relationship between a manufacturing system and a communication system. As shown in Figure 2, a communication system consists of four major elements including information source, communication channel, destination and noise source. These four elements in a communication system can also been mapped in a manufacturing system based on Figure 1. The corresponding mapped elements of a manufacturing system are also given in Figure 2.

Figure 2 shows how the key function of the channel is to reproduce the emitted message from the information source, as a received message at the destination. The emitted message (input of the channel) is a string of symbols from an alphabet created at the information source, while the received message (output of the channel) is the transmitted symbols received at the destination. The performance of the channel is affected by the noise source, which is evaluated by how well the channel output (received symbols) matches the channel input (emitted symbols).

For a mathematical representation of a (memoryless) discrete communication channel, a conditional probability matrix $P_{(S|D)}$ is often used. The matrix $P_{(S|D)}$ describes how each symbol is transmitted from the information source to the destination using the following matrix representation:

$$P_{(S|D)} = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0,M-1} \\ p_{10} & p_{11} & \cdots & p_{1,M-1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N-1,0} & p_{N-1,1} & \cdots & p_{N-1,M-1} \end{bmatrix} \tag{1}$$

Here, p_{ij} corresponds to the probability that symbol i in the channel inputs (the input alphabet set) is transmitted as symbol j in the channel outputs (output alphabet set). M and N are the number of symbols in the output alphabet and the input alphabet, respectively. Note that M and N can be any number, thus, allowing us to model a wide range of manufacturing processes. Equivalently, p_{ij} corresponds to the conditional probability defined as $p_{ij} = Prob\{Output = j \mid Input = i\}$ and satisfies the condition of $\sum_{\forall j} p_{ij} = 1.$ The probability of p_{ii} reflects the transmission quality of the channel when transmitting symbol i. On the other hand, the value of p_{ij} for $i \neq j$ represents the transmission errors and the total error rate for symbol i is equal to $q_{ii} = \sum_{\forall j \neq i} p_{ij} = 1 - p_{ii}.$ To extend the concept of transmission quality in a communication system into that in a manufacturing system, p_{ii} is interpreted as the process quality rate corresponding to the fraction of conforming part i produced by the process, while $q_{ii} = 1 - p_{ii}$ is interpreted as the process error rate corresponding to the fraction of non-conforming part i produced by the process. The values of p_{ii} and q_{ii} can be estimated based on historical production data.

The advantage of adopting the communication systems framework is that the well-developed metrics in communication systems (e.g., entropy, channel's capacity, transmission rate, etc.) can be further extended for analysing the input/output mapping relationship in a manufacturing system. The linkages of these performance metrics between a communication system and a manufacturing system will be discussed in the following section.

3 Performance metrics for manufacturing systems using information theory

Suppose the demanded types of products are denoted by the random variable D, which is defined over the set Γ_D containing the labels of all product types. In this way, the demand percentage of a product of type i can be represented as the frequency of producing the products of type i in the production process, which can be equivalently represented by the probability $P_i^D = Prob\{D = i\}$, where $D = i$ corresponds to the event that the next demanded part at the manufacturing process is a product of type i. Similarly, we use the random variable S defined over the set of Γ_S to represent the type of output products in a production process. In the same way, the probability of P_i^S represents the ratio of product type i in the outputs of the production process. Based on the concept of entropy defined in the information theory, a set of complexity measures will be defined as follows in a manufacturing system.

1 *Input demand complexity, measured by input entropy* $H(D)$. Based on information theory, it is known that the entropy of D reflects the average uncertainty of demand D, defined as:

$$H(D) = -\sum_{i=0}^{N-1} P_i^D \log P_i^D \tag{2}$$

Therefore, we can use the demand entropy of $H(D)$ to measure the input complexity induced by the demand variety of products.

2 *Output product complexity, measured by output entropy* $H(S)$. Similar to equation (2), the output entropy of $H(S)$ can be calculated as:

$$H(S) = -\sum_{j=0}^{M-1} P_j^S \log P_j^S \tag{3}$$

$H(S)$ is used to measure the output product complexity, reflecting the mixed ratio uncertainty of output products produced by the production process. For an ideal production process with no quality errors, the production mix ratio perfectly matches the demand mixed ratio, i.e., $P_i^S = P_i^D$, thus, leading to $H(S) = H(D)$.

3 *Process capability for handling demand complexity [simply called process capability for complexity (PCC)], measured by the mutual information* $I(D, S)$. Based on information theory, the mutual information of two random variables D and S can be defined as:

$$I(D, S) = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} P_{ij}^{DS} \log \frac{p_{ij}^{DS}}{p_i^D p_j^S} \tag{4}$$

where P_{ij}^{DS} is the joint probability distribution function of variables D and S, i.e., $P_{ij}^{DS} = Prob\{D = i, S = j\}$. Alternatively, $I(D, S)$ can also be represented as:

$$I(D, S) = H(S) - H\left(S \middle| D\right) \tag{5}$$

Here, $H(S \mid D)$ is the conditional entropy, which is defined as:

$$H\left(S|D\right) = -\sum\nolimits_{i=0}^{N-1}\sum\nolimits_{j=0}^{M-1} P_{ij}^{DS} \log P_{j|i}^{S|D} \tag{6}$$

where $P_{j|i}^{S|D}$ corresponds to the conditional probability $Prob\{S = j \mid D = i\}$. Therefore, $H(S \mid D)$ is a measure of the amount of uncertainty that, after knowing the value of input D, still remains in output S. As a result, under an ideal perfect production process, the output product complexity exactly matches the input demand complexity with $H(S) = H(D)$. In this case, we have $H(S \mid D) = 0$, which results in $I(D, S) = H(S) = H(D)$. Therefore, the process capability for handling demand complexity reaches its maximum. In contrast, under a completely random production process, output S and input D are independent of each other with $H(S \mid D) = H(S)$, which results in $I(D, S) = 0$. In this case, the process capability for handling demand complexity is zero. In a normal production process, $I(D, S)$ in equation (5) can be interpreted as the amount of information in output S predicted by input D, which reflects a general dependency relationship between output S and input D. Therefore, we will use $I(D, S)$ to reflect the process capability for handling demand complexity, simply called *PCC*. In this way, the *PCC* is defined in this paper as a metric to assess how well a production process can handle the demand variety of products in a manufacturing system.

4 *Process capacity for handling demand complexity (simply called MAX-PCC), measured by the channel's capacity C*. One of the most important concepts related to a communication channel is that of its capacity. The channel's capacity is a measure of how much information can reliably be transmitted through the channel. Formally, the channel capacity corresponds to the tightest upper bound of the rate at which a channel can reliably transmit information. Mathematically, the capacity C of a channel is defined as:

$$C = \max\nolimits_{P^D} I(D, S) \tag{7}$$

where the maximisation is obtained by varying the input probability vector P^D exclusively. By extending this concept into a production process in a manufacturing system, we can also define C as the *process capacity for handling demand complexity (simply called MAX-PCC)*.

5 *Normalised process capability for handling demand complexity (simply called NPCC), measured by the coefficient of constraint C_{SD}*. Since the input demand complexity $H(D)$ can arbitrarily inflate the operation complexity $I(D, S)$, the coefficient of constraint C_{SD} proposed in Coombs et al. (1970) is defined based on the normalised mutual information as:

$$C_{SD} = \frac{I(D, S)}{H(D)} \tag{8}$$

The coefficient of constraint C_{SD} is an index defined over the normalised range [0, 1], and is used to describe the normalised process capability for handling demand complexity.

Although several entropic-based metrics have been used in the literature for describing the manufacturing system complexity or demand complexity, this paper is at the first time to propose the metrics of *PCC* and *NPCC* to describe how the manufacturing process can handle the input demand variety. Specifically, *PCC* is used to evaluate different processes' performance under the same given demand variety, while *NPCC* is used when the demand varieties are different, especially to see how its capability will be changed over the different demand ratios. In this sense, *NPCC* can be further used to evaluate the robustness of a process under various demand ratios.

6 *Total process quality, measured by total transmission rate Q*. Based on the definition of matrix $P_{(S|D)}$, the total transmission rate Q of a communication channel is defined as:

$$Q = \sum_{i=0}^{N-1} P_i^D p_{ii} \tag{9}$$

In a production process, Q can represent the total percentage of conforming products produced by the production process, which is a weighted sum of the probabilities of producing the conforming products by their corresponding input mix ratios. Therefore, Q can be used as a metric for measuring the quality performance of a production process.

4 Characteristics of proposed metrics

In this section, we will use two examples to study the effect of different factors on the complexity of a manufacturing system. One example is used to study the effect of the total process quality on *NPCC*. A positive relationship is observed between the process quality performance and the amount of complexity that the plant is capable of handling. The other example is used to study the effect of increasing the number of product types in the demand, on the *NPCC* of a station. We will be able to observe how *PCC* is increased with the number of product types, while the amount of *NPCC* does not necessarily always increases with the number of product types in the demand.

4.1 Effect of process quality on PCC

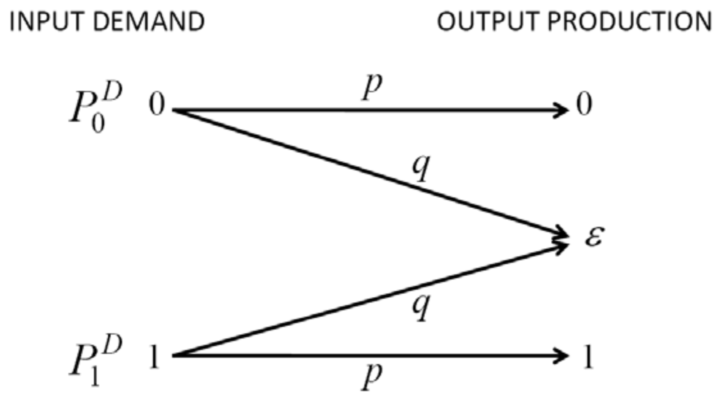
Two scenarios are provided in this subsection to study the effect of process quality on the *PCC* metric at a single work station: a two-type product scenario and a three-type product scenario, which will be discussed in detail as follows.

1 *Two types of products produced at a single operation station*. In order to consider the process quality error, the performance of the station is modelled by a symmetric erasure channel (Cover and Thomas, 2006) as shown in Figure 3, where the output symbol ε corresponds to non-conforming products. Assume the station produces the same quality rata for both types of products, i.e., $p_{11} = p_{22} = p$, the station can be modelled by a symmetric channel matrix as:

$$P_{(S|D)} = \begin{bmatrix} p & 0 & q \\ 0 & p & q \end{bmatrix}.$$

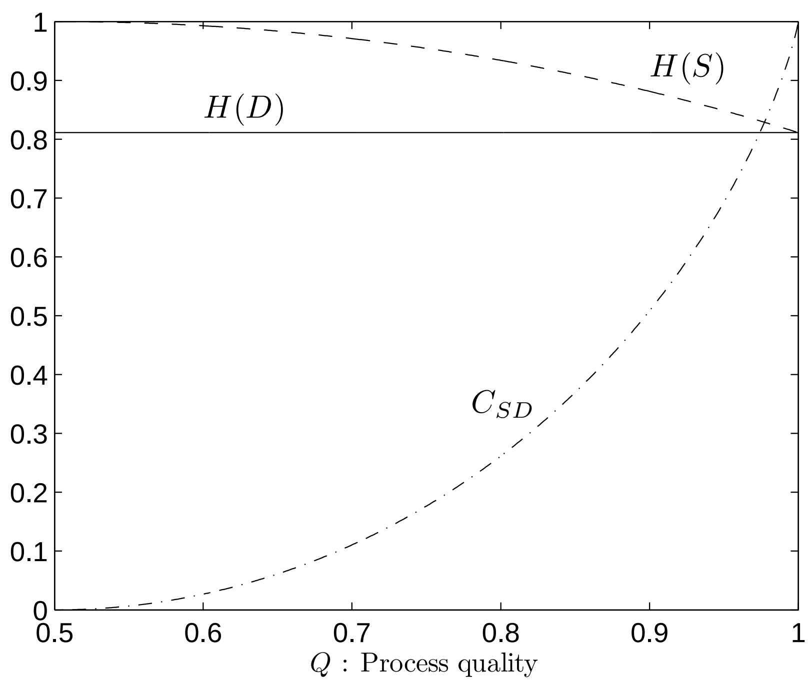
where the last column of q's shows the non-conforming fraction of parts produced at this station.

Figure 3 Representation of process quality for a station with a symmetric erasure channel



In order to study the effect of production quality, the input demand ratio will be set to $P_0 = 0.75$ and $P_1 = 0.25$, corresponding to Type 1 and Type 2 respectively. Also, production quality p is set within the range of (0.5, 1) because the quality rate below 0.5 is extremely unrealistic. Figure 4 shows the effect of the quality rate Q ($Q = p$ for a symmetric channel) on the *NPCC*. We have also included the input demand complexity and the output product complexity as references.

Figure 4 Input and output complexity and the *NPCC* VS process quality for two product types

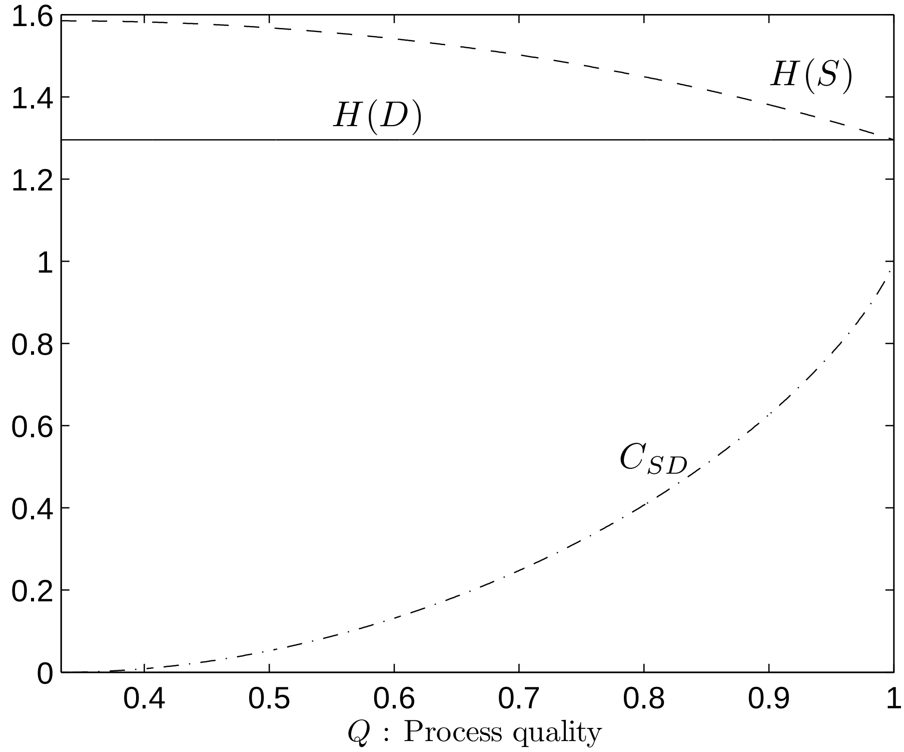


2 *Three types of products produced at a single operation station*. Similarly, the station can be modelled by a symmetric erasure channel as:

$$P_{(S|D)} = \begin{bmatrix} p & 0 & 0 & q \\ 0 & p & 0 & q \\ 0 & 0 & p & q \end{bmatrix}$$

The demand mix ratios of three types of products are set as $P_0 = 0.6$, $P_1 = 0.3$ and $P_2 = 0.1$. The total quality level Q will be set within the interval (1/3, 1). Figure 5 shows the effect of process quality Q on the *NPCC*, in which the input demand complexity and the output production complexity are also included as references.

Figure 5 Input and output entropy and *NPCC* VS process quality for three product types



From the above two-scenario analyses, it can be observed that the output product complexity monotonically decreases with the increase of the production quality and approaches the input complexity when production quality Q is close to one. This is because the uncertainty of the output products can only be as small as the uncertainty in the input demand. So, when the production quality $Q = 1$, the mix ratio of output parts exactly matches that of the input demand. The other conclusion is that the *NPCC* (C_{SD}) monotonically increases with the increment of quality Q. This conclusion is consistent with our intuition that if the process has a high process quality rate, it will have a high potential capability to handle the demand complexity under the same input mix ratio.

4.2 Effect of adding product types on PCC and NPCC

Let us consider a single station currently producing two types of products denoted as type 0 and type 1, respectively. We will study the effect of adding one, two and three new product types into the demand. In the case of adding one product type, it is assumed that the input demand rations P_1^D and P_2^D are equal. Similarly, when adding a fourth product type, it is assumed that $P_1^D = P_2^D = P_3^D$. The same assumption is made for the case with five product types.

Initially, the production quality with two product types is set to $p_i = 0.90$ for every product type *i*. Therefore, the total process quality of the plant is $Q = 0.90$. However, as the number of product types increases, the total process quality Q is expected to decrease exponentially, which we represent by the empirical rule:

$$p_i = \frac{1}{N^{0.15}} \tag{10}$$

where N is the number of product types in the demand. Figure 6 shows the plot of the production quality versus the number of choices.

Figure 6 Number of choices VS probability of producing a conforming product

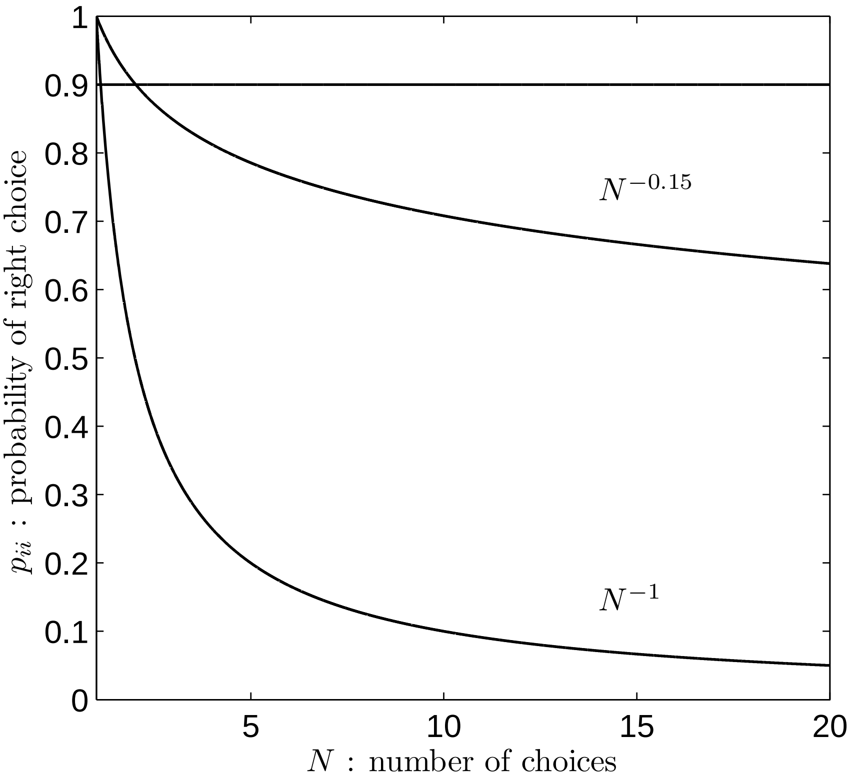


Figure 7 Effect of adding product types on the choice complexity

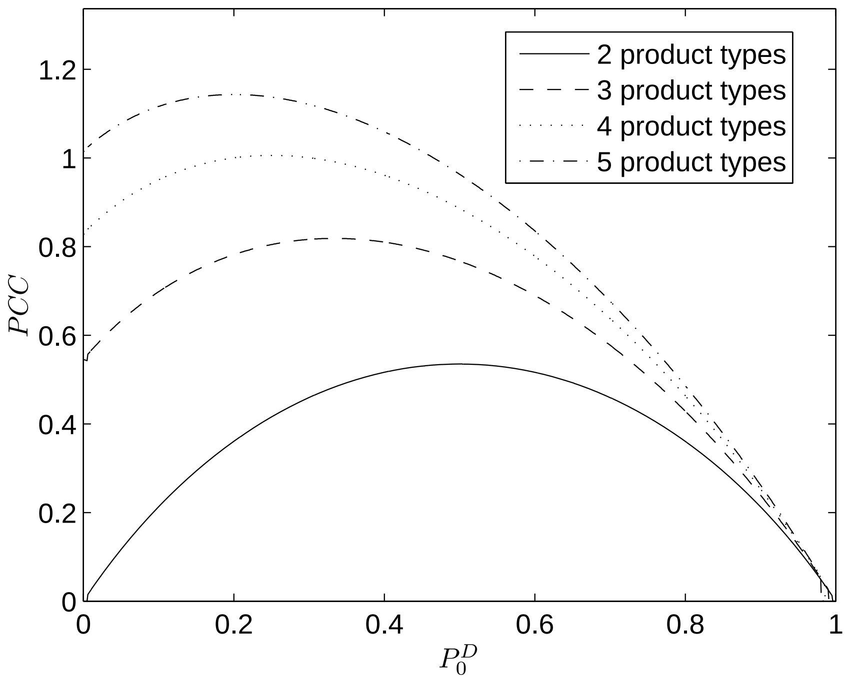


Figure 8 Effect of adding product types on the *NPCC*

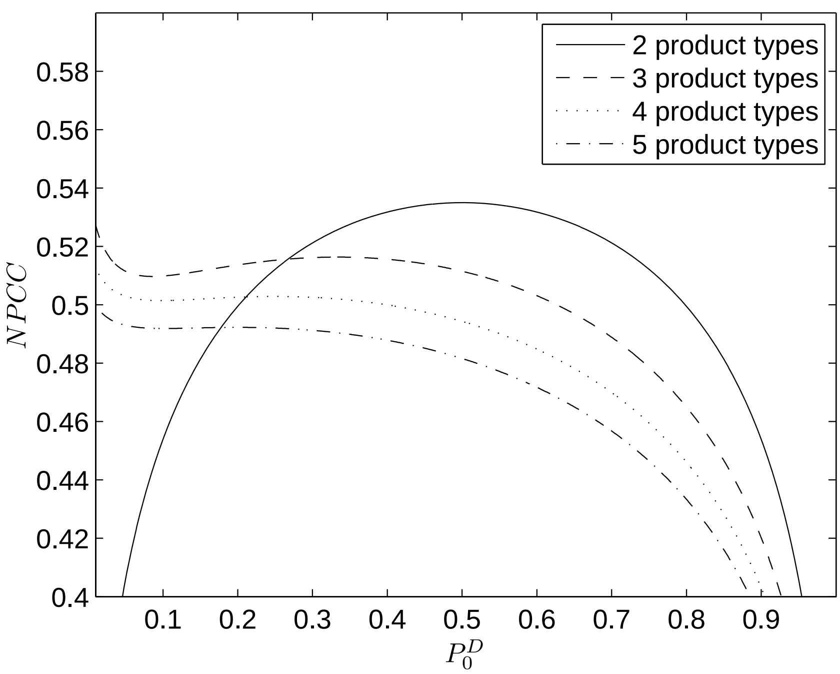


Figure 7 shows the positive effect of adding more product types to the demand on the *PCC* measured by the mutual information. From this figure, it can be seen that as the input demand ratio for product type 0, P_0^D, approaches 1, $I(D, S)$ under all four cases converges to zero. The reason for this is that as P_0^D gets closer to one, the input complexity also approaches zero, and hence, there is no information to be shared between the inputs and the outputs of the system.

Furthermore, Figure 8 shows the effect of adding more product types on the *NPCC*. It can be approximately seen that when $P_0 < 0.26$, C_{SD}, under the three-product-type case, is greater than that under the two-product-type case. In contrast, when $P_0 > 0.26$, C_{SD}, under the two-product-type case, is greater than that under the three-product-type case. Similar results can be observed for all other scenarios as well.

5 Metrics for a two-station process

5.1 Process description

This section is used to illustrate how the metrics of *PCC* and *NPCC* are affected by three different layouts of a production process, which is designed to produce two different parts used as two components in a final product. Each part (component) is designed to have two varieties, thus, resulting in a total of four possible combinations of two parts to be used in final products. Each possible part combination has a different demand percentage represented by the following vector:

$$P^D = \begin{bmatrix}\pi_{00} & \pi_{01} & \pi_{10} & \pi_{11}\end{bmatrix} \tag{11}$$

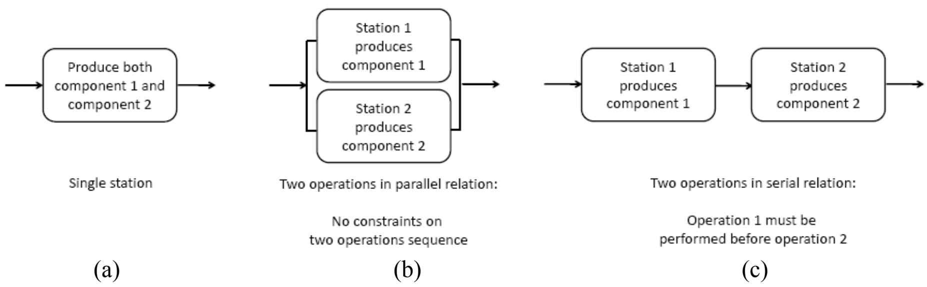
where π_{ij} corresponds to the mix ratio of a product consisting of part 1 of type *i* and part 2 of type *j*. In our example, the mix ratio of the demand is set to:

$$P^D = \begin{bmatrix}0.40 & 0.30 & 0.20 & 0.10\end{bmatrix}$$

The input demand complexity can be computed based on equation (2), yielding $H(D) = 1.8464$.

Figure 9 shows a diagram of the three possible production process configurations considered in this example.

Figure 9 Three different production process configurations, (a) single station (b) two operations in parallel relation* (c) two operations in serial relation**



Notes: *No constraints on two operations sequence
**Operation 1 must be performed before operation 2

In Layout (a), as shown in Figure 9, a single station is used to produce both part 1 and part 2 by using a single machine with the reconfigurable tooling capability. The production quality rate for each possible combination of parts can be assigned individually.

In Layout (b), as shown in Figure 9, two separate machines are used at two stations to produce part 1 and part 2 separately in a parallel configuration. In this paper, the parallel configuration means that there is no constraint on the operation sequence between operation 1 and operation 2, i.e., part 1 can be produced at station 1 either before or after part 2 is produced at station 2. Since a final product consists of both components (part 1 and part 2), the amount of final conforming products corresponds to the minimum amount of each of the two conforming components produced in this process. Therefore, the probability of the final product quality is equal to the minimum conforming fraction for producing part 1 and part 2.

In Layout (c), as shown in Figure 9, two separate machines are used at two stations to produce part 1 and part 2 separately, but in a serial configuration. Different from the parallel configuration in Layout (b), a serial configuration indicates a constraint on the operation sequence for producing part 1 and part 2, i.e., operation 1 for producing part 1 must be performed before operation 2 for producing part 2. For example, some features of part 1 may be used as the tooling fixture references at operation 2 for producing part 2. Therefore, the overall conforming fraction of a final product is the product of two conforming fractions associated with both parts.

It is worthwhile to clarify that the parallel and serial configurations shown in Figure 9 are used to distinguish whether the selected process has a constraint on the operation sequence for producing part 1 and part 2 when using different machines. Specifically, the parallel configuration means part 1 and part 2 can be produced separately by two independent operations at two different stations. In contrast, the serial configuration requires producing part 2 after producing conforming part 1. The selected two machines used in the parallel configuration may be different from those used in the serial configuration.

5.2 Process modelling

Each process layout yields a different output product complexity $H(S)$ and PCC due to the constraints on the operation sequences that the selected production process must follow. In this example, all the stations will be modelled by a general erasure channel, as discussed below.

5.2.1 Layout (a) model

Station 1 can be represented by the following matrix:

$$P_{(S|D)} = \begin{bmatrix} p_{00}^{*} & 0 & 0 & 0 & q_{00}^{*} \\ 0 & p_{01}^{*} & 0 & 0 & q_{01}^{*} \\ 0 & 0 & p_{10}^{*} & 0 & q_{10}^{*} \\ 0 & 0 & 0 & p_{11}^{*} & q_{11}^{*} \end{bmatrix} \tag{12}$$

where p_{ij}^{*} corresponds to the probability of producing conforming products with part 1 of type *i* and part 2 of type *j*. Since $P_{(S|D)}$ is a probability matrix, $q_{ij}^{*} = 1 - p_{ij}^{*}$. The output mix ratio is obtained by:

$$P^S = \left[p_{00}^{*}\pi_{00} \quad p_{01}^{*}\pi_{01} \quad p_{10}^{*}\pi_{10} \quad p_{11}^{*}\pi_{11} \quad q_{00}^{*}\pi_{00} + q_{01}^{*}\pi_{01} + q_{10}^{*}\pi_{10} + q_{11}^{*}\pi_{11} \right] \quad (13)$$

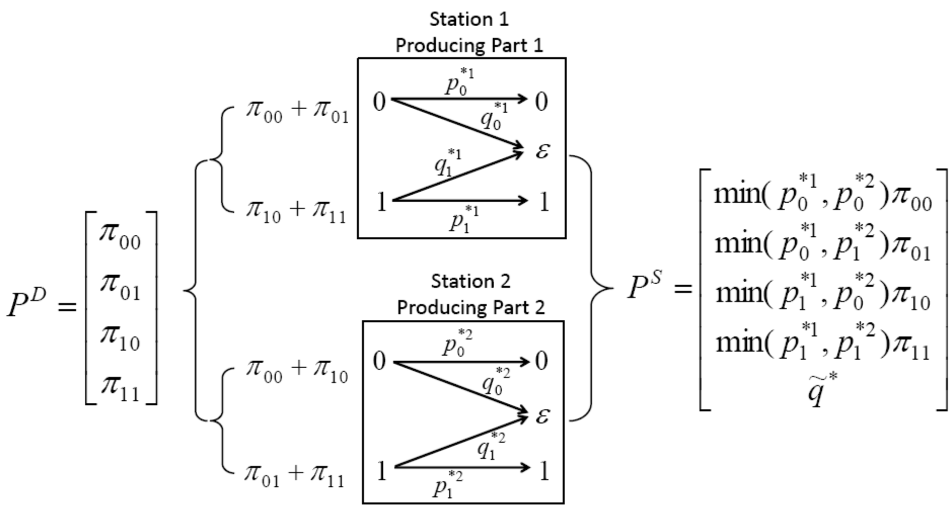
5.2.2 Layout (b) model

The model structure of Layout (b) is represented by the diagram in Figure 10. The probability matrix representing station $i = 1, 2$ is given by:

$$P_{i,(S|D)} = \begin{bmatrix} p_0^{*i} & 0 & q_0^{*i} \\ 0 & p_1^{*i} & q_1^{*i} \end{bmatrix} \quad (14)$$

where p_j^{*i} corresponds to the probability of producing conforming products with part *i* of type *j*. It should be kept in mind that part *i* is only produced at station *i*. As before, q_j^{*i} corresponds to the probability of producing non-conforming products with part *i* of type *j*.

Figure 10 Layout (b) model



The output mix ratio after station 1's operation is:

$$P_1^S = \left[p_0^{*1}(\pi_{00} + \pi_{01}) \quad p_1^{*1}(\pi_{10} + \pi_{11}) \quad q_0^{*1}(\pi_{00} + \pi_{01}) + q_1^{*1}(\pi_{10} + \pi_{11}) \right] \quad (15)$$

while the output mix ratio after station 2 is:

$$P_2^S = \left[p_0^{*2}(\pi_{00} + \pi_{10}) \quad p_1^{*2}(\pi_{01} + \pi_{11}) \quad q_0^{*2}(\pi_{00} + \pi_{10}) + q_1^{*2}(\pi_{01} + \pi_{11}) \right] \quad (16)$$

The production process formed by these two stations together can be modelled by the matrix:

$$P_{(S|D)} = \begin{bmatrix} \min\left(p_0^{*1}, p_0^{*2}\right) & 0 & 0 & 0 & \max\left(q_0^{*1}, q_0^{*2}\right) \\ 0 & \min\left(p_0^{*1}, p_1^{*2}\right) & 0 & 0 & \max\left(q_0^{*1}, q_1^{*2}\right) \\ 0 & 0 & \min\left(p_1^{*1}, p_0^{*2}\right) & 0 & \max\left(q_1^{*1}, q_0^{*2}\right) \\ 0 & 0 & 0 & \min\left(p_1^{*1}, p_1^{*2}\right) & \max\left(q_1^{*1}, q_1^{*2}\right) \end{bmatrix} \quad (17)$$

The output system mix ratio can be calculated by:

$$P^S = \left[\min\left(p_0^{*1}, p_0^{*2}\right)\pi_{00} \ \min\left(p_0^{*1}, p_1^{*2}\right)\pi_{01} \ \min\left(p_1^{*1}, p_0^{*2}\right)\pi_{10} \ \min\left(p_1^{*1}, p_1^{*2}\right)\pi_{11} \ \tilde{q}^*\right] \quad (18)$$

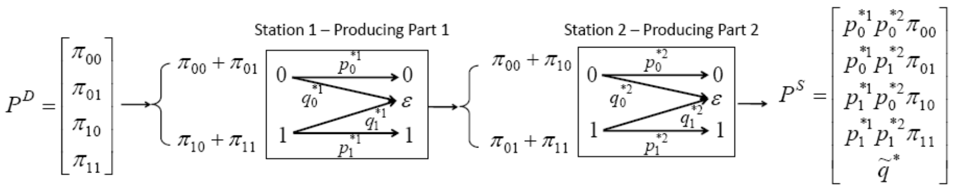
where

$$\tilde{q}^* = \max\left(q_0^{*1}, q_0^{*2}\right)\pi_{00} + \max\left(q_0^{*1}, q_1^{*2}\right)\pi_{01} + \max\left(q_1^{*1}, q_0^{*2}\right)\pi_{10} + \max\left(q_1^{*1}, q_1^{*2}\right)\pi_{11}.$$

5.2.3 Layout (c) model

The model structure of Layout (c) is represented by the diagram in Figure 11. Layout (c) corresponds to the cascaded communication channels configuration (Silverman, 1955), and hence, the extensive literature dedicated to their study could be used in analysing this layout.

Figure 11 Layout (c) model



The probability matrix representing station $i = 1, 2$ is given by:

$$P_{i,(S|D)} = \begin{bmatrix} p_0^{*i} & 0 & q_0^{*i} \\ 0 & p_1^{*i} & q_1^{*i} \end{bmatrix} \quad (19)$$

where p_j^{*i} corresponds to the probability of producing conforming products with part i of type j. The output mix ratio after station 1 is:

$$P_1^S = \left[p_0^{*1}\left(\pi_{00} + \pi_{01}\right) \quad p_1^{*1}\left(\pi_{10} + \pi_{11}\right) \quad q_0^{*1}\left(\pi_{00} + \pi_{01}\right) + q_1^{*1}\left(\pi_{10} + \pi_{11}\right)\right] \quad (20)$$

while the output mix ratio after station 2 is:

$$P_2^S = \left[p_0^{*1} p_0^{*2}\pi_{00} + p_1^{*1} p_0^{*2}\pi_{10} \quad p_0^{*1} p_1^{*2}\pi_{01} + p_1^{*1} p_1^{*2}\pi_{11} \quad \tilde{q}_2^*\right] \quad (21)$$

where

$$\tilde{q}_2^* = \left(1 - p_0^{*1} p_0^{*2}\right)\pi_{00} + \left(1 - p_1^{*1} p_0^{*2}\right)\pi_{10} + \left(1 - p_0^{*1} p_1^{*2}\right)\pi_{01} + \left(1 - p_1^{*1} p_1^{*2}\right)\pi_{11}$$

The matrix representing the system as a whole is:

$$P_{(S|D)} = \begin{bmatrix} p_0^{*1}p_0^{*2} & 0 & 0 & 0 & q_0^{*1} + p_0^{*1}q_0^{*2} \\ 0 & p_0^{*1}p_1^{*2} & 0 & 0 & q_0^{*1} + p_0^{*1}q_1^{*2} \\ 0 & 0 & p_1^{*1}p_0^{*2} & 0 & q_1^{*1} + p_1^{*1}q_0^{*2} \\ 0 & 0 & 0 & p_1^{*1}p_1^{*2} & q_1^{*1} + p_1^{*1}q_1^{*2} \end{bmatrix} \tag{22}$$

The output mix ratio of the system is:

$$P^S = \begin{bmatrix} p_0^{*1}p_0^{*2}\pi_{00} & p_0^{*1}p_1^{*2}\pi_{01} & p_1^{*1}p_0^{*2}\pi_{10} & p_1^{*1}p_1^{*2}\pi_{11} & \tilde{q}^* \end{bmatrix} \tag{23}$$

where

$$\tilde{q}^* = \left(q_0^{*1} + p_0^{*1}q_0^{*2}\right)\pi_{00} + \left(q_0^{*1} + p_0^{*1}q_1^{*2}\right)\pi_{01} + \left(q_1^{*1} + p_1^{*1}q_0^{*2}\right)\pi_{10} + \left(q_1^{*1} + p_1^{*1}q_1^{*2}\right)\pi_{11}.$$

5.3 Process conditions and analysis results

In Layout (a), a single station is used to produce both part 1 and part 2 by using a single machine with the reconfigurable tooling capability. It is assumed that the same conforming probability of $p_{ij}^* = 0.85$ is obtained for the four different final product combinations. Table 1 shows the detail demand mix ratio and the associated conforming probability.

Table 1 Demand and conforming percentages for single station [Layout (a)]

	Single station			
	Part 1 Type 0 *Part 2 Type 0*	*Part 1 Type 0* *Part 2 Type 1*	*Part 1 Type 1* *Part 2 Type 0*	*Part 1 Type 1* *Part 2 Type 1*
Demand	40%	30%	20%	10%
Conforming percentage	0.8500	0.8500	0.8500	0.8500

In Layout (b) and Layout (c), the conforming probabilities of each station will be selected to reflect the dependency between the quality and the demand ratio for each specific combination of part *i* and type *j*. Specifically, for the highly demanded part/type, a machine with a higher conforming probability is used, as shown in Table 2.

Table 2 Demand and conforming percentages for Station 1 and Station 2 [Layout (b) and Layout (c)]

	Station 1 (Part 1)		*Station 2 (Part 2)*	
	Type 0	*Type 1*	*Type 0*	*Type 1*
Demand	70%	30%	60%	40%
Conforming percentage	0.9750	0.8500	0.9500	0.9000

Table 3 shows the analysis results for each of the above three possible layouts. It can be seen that the less restrictive Layout (b) with the parallel configuration has the highest *NPCC* among these three layouts. Additionally, the metric *NPCC* and the total quality *Q* are positively related as intuitively expected.

Table 3 Metrics under three process configurations

	Input complexity $H(D)$	*Output complexity* $H(S)$	*PCC* $I(D, S)$	*NPCC* C_{SD}	*Process quality* Q
Layout (a)	1.8464	2.1793	1.5695	0.8500	0.8500
Layout (b)	1.8464	2.1000	1.6618	0.9000	0.9050
Layout (c)	1.8464	2.1261	1.5933	0.8629	0.8718

6 Exemplary use of methodology

The following example is used to illustrate one of the possible applications of the proposed metrics *PCC* and *NPCC*. We will consider a single station that produces three different types of products. Based on historical data, we know that product type 0 is easier to be produced than the other two types of products, i.e., the quality rate of product type 0 is estimated as p_{00} = 0.90, while for the other products, the quality rates are estimated as p_{11} = 0.85 and p_{22} = 0.80 respectively. Currently, the production line is used to produce twice as many products of type 0 as that of type 1 and type 2 together. Equivalently, the input demand ratios are estimated as $P_0^D = .67$ and $P_1^D = P_2^D = 0.165$, respectively. At these levels, the total process quality of the plant is Q = 0.8752. The *NPCC* at such settings can be computed as C_{SD} = 0.5288. The plant is interested in achieving the maximum amount of *NPCC* by adjusting the mix ratio of the demand under the constraint of the total process quality at least equal to 0.875.

Figure 12 Total process quality Q

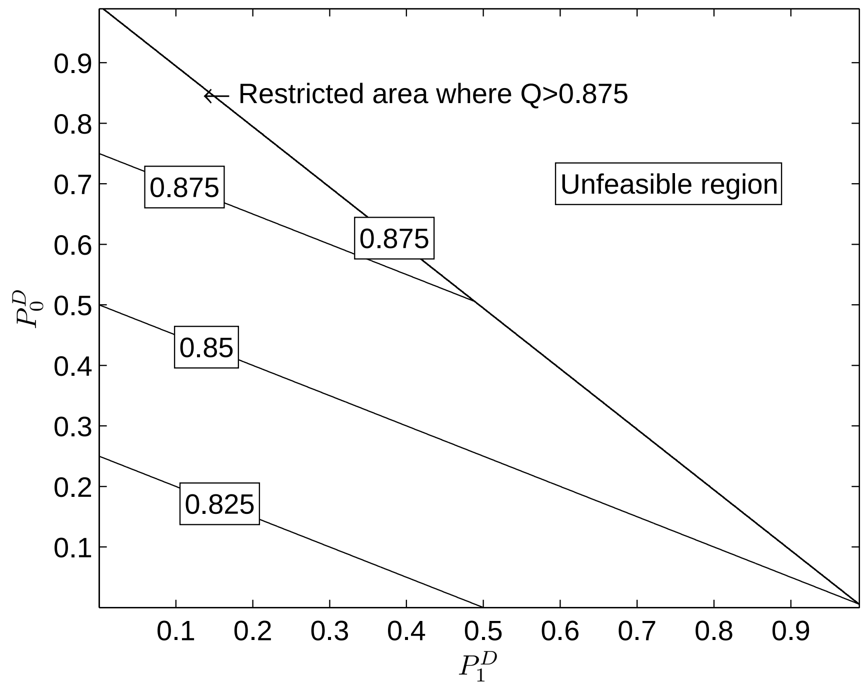


Figure 12 presents the contour plot of the process quality Q under all the possible combinations of the demand ratios P_0^D, P_1^D and P_2^D. In this plot, the unfeasible region corresponds to the combinations of the demand ratios where $P_0^D + P_1^D + P_2^D > 1$.

Table 4 summarises all the analysis results under different criteria. The first column of Table 4 provides all indexes under the initial settings of the input demand ratios for three types of products. The second column contains the optimal input demand ratios that maximises C_{SD} yielding the maximum value of $C_{SD}^* = 0.5934$. This result can also be obtained based on the contour plot of C_{SD} as shown in Figure 13, in which the maximum point (the dot point) is achieved at the condition of $P_0^{D*} = 0.5381$, $P_1^{D*} = 0.4561$ and $P_2^{D*} = 0.0058$. Again, the unfeasible region corresponds to demand probabilities values where $P_0^D + P_1^D + P_2^D > 1$.

Table 4 Input probabilities with corresponding C_{SD} and Q

	Initial setting	*Optimal criteria*		
		Max C_{SD}^*	*Max* Q^*	*Max* C_{SD}^* *s.t.:* $Q > 0.875$
P_0^D	0.670	0.5381	1	0.5381
P_1^D	0.165	0.4561	0	0.4561
P_2^D	0.165	0.0058	0	0.0058
C_{SD}	0.5288	0.5934	NA	0.5934
Q	0.8752	0.8766	0.9000	0.8766

In the third column of Table 4, it can be seen that the total process quality Q is maximised under the condition of $P_0^D = 1$. This solution is expected since the total process quality Q surface shown in Figure 12 is linear in all its variables, thus, the optimal value occurs at the upper left corner of Figure 12.

Finally, the fourth column results of Table 4 provide the optimal input demand ratio that maximises C_{SD}, subjected to the process quality level $Q' \geq 87.5\%$. The restricted area defined by $Q > 87.5\%$ is also labelled in Figure 12. The value of C_{SD}, satisfying the constraint of $Q' \geq 87.5\%$ area is plotted in Figure 14, where the location of the maximum has been marked with a dot.

From Table 4, it can be seen that the restricted maximum is the same as the unrestricted one since the total process quality under the unrestricted case is above 87.5%. If only considering the total process quality and the *NPCC* as the decision criteria, the solution suggests to completely eliminate product type 2 from the demand and recalibrate the mix ratio of the other two types of products to achieve the maximum *NPCC* at a satisfying process quality rate. The reason for this is that the conforming rate of type 2 product is the lowest among the three types of products. Consequently, the production of type 2 products decreases the total process quality at the plant, while not significantly increases the *NPCC* of the plant.

Figure 13 *NPCC* C_{SD}

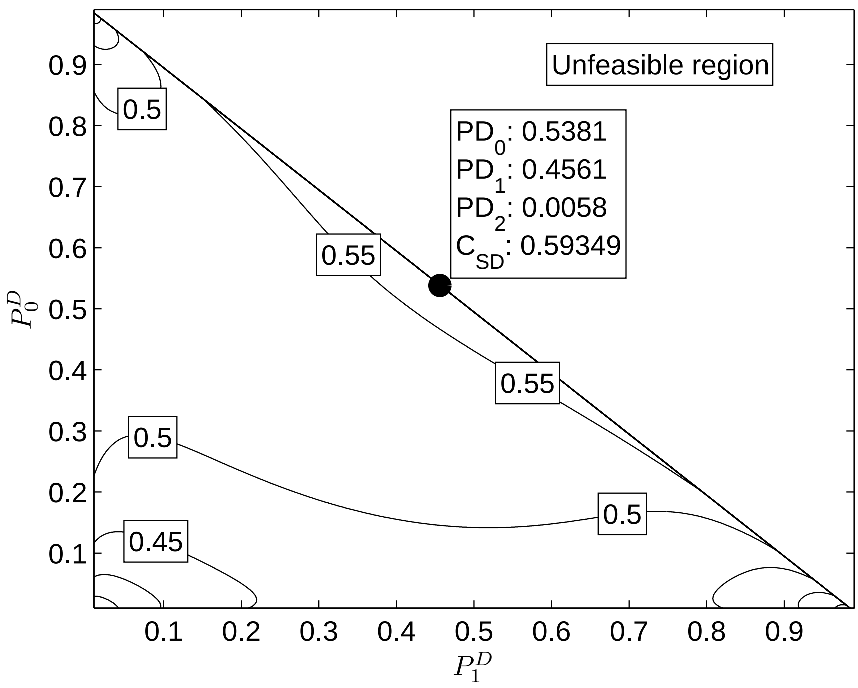
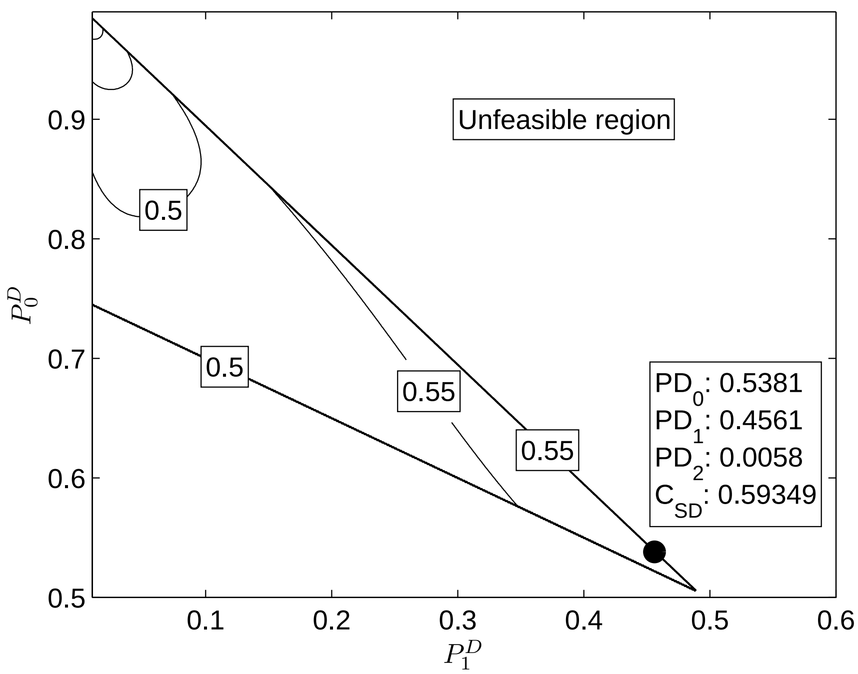


Figure 14 Restricted coefficient of constraint C_{SD}



7 Conclusions and future work

This paper is at the first time to present a new modelling framework for measuring the handled complexity of a manufacturing system based on a communication system framework, in which the linkages of input/output relationship between these two systems are discussed. Different from the literature, this paper is at the first time to consider production quality into complexity metrics. In fact, when considering process quality, we see a divergence between what is demanded and what is produced. A measure of divergence between the input mix ratio and the output mix ratio can be interpreted as a measure of the amount of complexity that the manufacturing process cannot deliver. To measure the divergence between the complexity in the demand and the complexity in the delivered products, the metrics for *PCC* and for *NPCC* are defined, which are used to assess the process capability and normalised process capability for handling input complexity, respectively. Several examples are given in the paper to illustrate different aspects of the proposed metrics. Moreover, the effects of production quality and multiple stations' layout configuration are also investigated. Finally, an example on the use of the proposed methodology is illustrated to show how the defined metrics, the *PCC* and the *NPCC* are used to obtain the maximum process capability under the constrained total process quality.

It should be clarified that this paper only focuses on defining complexity metrics for measuring PCC that are induced by the demand mix ratio. Future research will explore how to measure the process capability for handing operational complexity by further considering the frequency of changeover or the batch size of product varieties. For this purpose, the predictability of the product sequences will be further studied and included into the measure of input complexity in addition to the demand mix ratio. In addition, although a single symbol ε is used in the paper to represent the total non-conforming products, the model can be further extended by using ε_{ij} to represent different defects, which can also be handled by using the communication systems theory. In this way, we can further include other aspects of the manufacturing systems, such as different scrap costs, rewards, root causes, etc., into the model.

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