Modeling and Analysis of Operator Effects on Process Quality and Throughput in Mixed Model Assembly Systems

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With the increase of market fluctuation, assembly systems moved from a mass production scheme to a mass customization scheme. Mixed model assembly systems (MMASs) have been recognized as enablers of mass customization manufacturing. However, effective implementation of MMASs requires, among other things, a highly proactive and knowledgeable workforce. Hence, modeling the performance of human operators is critically important for effectively operating these manufacturing systems. But, certain cognitive factors have seldom been considered when it comes to modeling process quality of MMASs. Thus, the objective of this paper is to introduce an integrated modeling framework by considering the factors—both intrinsic (such as work experience, mental deliberation time, etc.) and extrinsic (such as task complexity)—that affect the operator's performance. The proposed model is justified based on the findings presented in the psychological literature. The effect of these factors on process operation performance is also investigated; these performance measures include process quality, throughput, and process capability in regard to handling complexity induced by product variety in MMASs. Two examples are used to demonstrate potential applications of the proposed model. [DOI: 10.1115/1.4003793]

Keywords: choice complexity, mixed assembly process, operator performance, quality conforming rate, thinking time, throughput

1 Introduction

With the increase of market fluctuation, assembly systems moved from a mass production scheme to a mass customization scheme. The intent of mass customization is to provide customers with products that are close to their specific demands by producing a wide range of product varieties. At the same time, mass customization keeps costs low, allowing customized products to remain affordable. Mixed model assembly systems (MMASs) have been recognized as enablers of mass customization manufacturing. However, it has been shown by both empirical and simulation results [1–3] that increasing the variety of automobile production has a significantly negative impact on the performance of the MMAS on process quality and productivity. Such an impact affects not only the complexity of system configurations and operations but also the requirements placed on the operators, who must develop skills for handling the multiple tasks needed to produce product variants. The role of operator's performance is especially critical in MMASs requiring intensive labor operations (e.g., automobile final general assembly [4]) and personalized product manufacturing. Therefore, in order to successfully implement a mass customization scheme, MMASs should be appropriately designed and operated by considering the operator's capability.

Modeling human operators' performance in an assembly system is a challenging research task, mainly because the factors involved are difficult to identify, define, and measure. Nevertheless, the inclusion of human operators' modeling is necessary for accurate system performance predictions. In this spirit, Bernhard and Schilling [5] asserted that inaccuracy on the simulation results is particularly apparent when modeling manufacturing systems with a high proportion of manual operations. Baines et al. [6] further indicated that the production rate obtained from a simulation without considering the operator's effect is usually higher than the system's actual rate of the real system and that, as a result, a better accuracy in the simulation results could be achieved by including important factors affecting the operators' performance.

The factors that affect the performance of operators can be classified into two categories: intrinsic and extrinsic. Intrinsic factors include factors such as the operator's age, working skills, experience, and so forth. Extrinsic factors include factors such as task difficulty and the number of choices that must be made in a given manufacturing task. Most models of human behavior and performance consider some of these factors.

Some research has been conducted to study the operator's effect in relation to process performance. For example, Fine [7] developed a high-level model to describe the relationship of production cost with the factors of operator's learning skills and quality improvement efforts. In Ref. [4], a model was proposed to study the effect of an operator's age on his or her performance, which was measured as the amount of time required to finish a task. Furthermore, Wang and Hu [8] considered the operator's fatigue due to product variety and studied its effect on system performance. None of these studies, however, have focused on how to adjust the production operations accordingly to the operator's performance (e.g., designing a task cycle time according to the operator's performance under different levels of task difficulty and working experience).

In MMASs, the number of product types and their mixed ratios varies among different operation stations. Therefore, the operator's performance will be affected by the choice task complexity and the amount of experience required at the specific station. The allocation of a task cycle time is important to ensure that the

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operator will have sufficient mental deliberation thinking time to make the correct choice of parts, tools, and activities for task completion. In contrast, an excessive allocation of a task cycle time wastes production time and decreases the production throughput. Therefore, the objective of this study is to develop an integrated model for effectively characterizing the operator's performance and analyzing its effect on process quality and throughput in MMASs; this will be accomplished by considering both intrinsic and extrinsic factors in the model. Specifically, the study will examine the operator's working experience and mental deliberation thinking time as intrinsic factors and the choice task complexity due to the product variety as an extrinsic factor. We also present examples to show how to optimally allocate individual cycle time at each station to maximize the overall quality conforming rate, throughput, and process capability for handling complexity (PCC), which consider the operator's effect.

The first intrinsic factor we consider is the operator's experience. Traditionally, an operator's experience for performing a particular task refers to that operator's autonomous learning ability due to the repetitive execution of that task [9]. While literature on learning curves abounds (e.g., Refs. [10,11]), there is limited research that addresses the effect of experience on quality conforming performance. Most of these existing models are based on the exponential learning curve, which was developed by Wright [12]. Later, Fine [7] studied quality improvement and learning capabilities together and analyzed their interactive effects on production costs. Kini [13] extended such ideas by considering the impact of nonconforming units in the learning process. A model addressing quality improvement as a result of the learning effect was also proposed by Li and Rajagopalan [14]. In their model, the learning effect was caused by the accumulation of knowledge stemming from production and process improvements. In this study, we will focus only on the autonomous learning performance through the repetitive execution of a task in a MMAS. An exponential model will be used to describe the quality conforming performance in terms of the cumulative production cycles performing the task. This will be discussed in detail in Sec. 2.

The second intrinsic factor we will examine is the operator's mental deliberation thinking time. Intuitively, the performance of a task is positively related to the time available to perform the task. In other words, there is a tradeoff between speed and accuracy in task performance, a relationship widely accepted in literature [15–17]. A number of research studies have supported these findings. For example, Schouten and Bekker [18] studied how the amount of time available affected the correct rate of recognizing an auditory cue. In another experiment conducted by Pachella and Pew [19], a subject was presented with various configurations of four lights and had to respond with four of his or her fingers accordingly; as the task was performed, the response time was recorded. In these two experiments, the relationship between accuracy and response time was found to be well modeled by an exponential curve, as reported by Pew [20].

More elegant attempts to model this tradeoff have been more recently proposed, all describing the exponential relationship between speed and accuracy. For example, in decision-making tasks, diffusion process models [21–23] have gained increasing acceptance over the past few decades, mainly because of their ability to effectively describe the tradeoff between speed and accuracy.

In this study, we assume that an operator requires some mental deliberation thinking time in order to achieve a satisfying quality conforming rate. Specifically, the operator's mental thinking time is referred to as the time employed by the operator to accomplish cognitive tasks that are essential to add value to the process quality performance. In this context, value-adding cognitive tasks may include recognizing what product variant to produce, selecting the right tools, performing mental evaluations, and acquiring production process environmental awareness. Based on existing research, we will model the effect of thinking time on the process quality conforming rate by an exponential function. This will be discussed in detail in Sec. 2.

The third operator's factor considered in this study is the extrinsic factor of choice task complexity due to product variety in a MMAS. It is widely known that there is a strong connection between the theoretical properties of information theory's entropy [24] and experimental findings in cognitive science describing the mental workload imposed on subjects when making a choice among multiple alternatives. Based on a classical experiment, Woodworth and Schlosberg [25] and Hick [26] discovered that if all the choices are equally likely to be chosen, the performance of the operator is inversely proportional to the logarithmic of the number of choices. This finding was revolutionary to mathematical cognitive science.

Later, Hyman [27] extended this idea to the unequally likely choices' scenario. The result became known as the Hick–Hyman law, which describes a negative relationship between the performance of an individual and the information entropy of the choices. Recently, in a mixed model assembly system environment, Zhu et al. [28] and Hu et al. [29] proposed the use of information entropy as a measure of the choice task complexity due to the demand variety; this was called the "operator choice complexity." In this study, the operator choice complexity index is used to quantify the effect of the task complexity on the operator's performance in performing assembly operations in a MMAS. Since entropy is essentially a weighted average of the logarithmic of the probability of each outcome of an event, there is an exponential relationship between the event outcome proportions and its entropy. This result motivates us to consider an exponential model to describe the relationship between the process quality performance and choice task complexity. This will be discussed in detail in Sec. 2.

The organization of the rest of the paper is as follows: Sec. 2 describes the proposed mathematical model used to characterize the operator's performance in a MMAS. In Sec. 3, the proposed model is used to analyze the effect of the operator's factors on the throughput of an assembly system. The analysis is first discussed under a single-station scenario and then under a multiple-station assembly line scenario. Two illustrative examples are given in Sec. 4 to show possible applications of the proposed model. Finally, Sec. 4 provides the conclusions and indicates possible directions for future research.

2 Modeling and Analysis of Operators' Factors

2.1 Proposed Integrated Modeling Framework and Mathematical Models. Figure 1 shows the proposed integrated modeling framework, which describes how an operator's intrinsic (working experience and mental deliberation thinking time) and extrinsic (choice task complexity due to part mixed ratio) factors affect the process quality. Favorable levels of the operator's factors will improve the operator's cognitive performance and thus improve the process quality conforming rate above its *worst quality level*. Here, the term worst quality level refers to the lowest quality conforming rate that results from the most adversarial levels of the operator's factor scenario such as an unskillful operator performing the most complex task with no deliberation thinking time. Moreover, Fig. 1 also shows that the mental thinking time affects the production cycle time, while the throughput is affected by both the process quality and the production cycle time. Based on this proposed integrated framework, it is important to note that an increase in the operator's thinking time allocation will increase the process quality performance as well as the production cycle time, both oppositely affecting the production rate. Therefore, a mathematical model is needed to quantitatively describe the contrary effect of thinking time on process quality and production cycle time. This will be described in Sec. 3 in greater detail.

Research has shown that a mathematical model can be defined

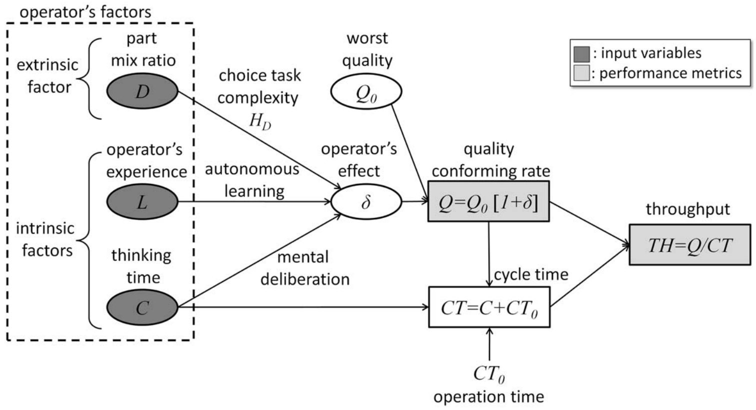

Fig. 1 Proposed integrated modeling framework

by an exponential function that is used to describe the operator's effect of a single factor F (e.g., D, L, and C, as shown in Fig. 1) on the process quality conforming rate as

$$Q(F) = Q_0[1 + \delta(F)] \tag{1}$$

where Q_0 is the lowest process quality conforming rate, which corresponds to the worst quality performance that results from the most adversarial operator's factor scenario. $\delta(F)$ is the percentage of the improved quality conforming rate under a more favorable operator's factor scenario F than that of the worst scenario. Therefore, the larger the factor F is, the more quality improvement of $\delta(F)$ is expected. Based on general ideas found in psychophysics literature [30] where an exponential relationship is proposed to model the effect between the magnitude of a physical stimuli and its human's perception, the effect of the operator's factors will be characterized by an exponential relationship. Thus, the following exponential model is proposed for $\delta(F)$:

$$\delta(F) = \Delta_M(1 - e^{-\beta F}) \tag{2}$$

where Δ_M is the upper limit of the quality conforming rate that can be maximally improved under the most favorable operator's factor scenario and β is a positive parameter defining the increase rate of quality with respect to factor F. The most favorable factor scenario corresponds to a very experienced operator performing the simplest task with sufficient mental deliberation thinking time. In this way, $\delta(F)$ is a convex increasing bounded function, which satisfies $\delta(F)=0$ (corresponding to the worst scenario) and $\delta(F) \rightarrow \Delta_M$ as $F \rightarrow \infty$ (corresponding to the most favorable scenario). Furthermore, the increasing rate of $\delta(F)$ is further attenuated as the value of F increases, which is a desired characteristic to reflect the limited operator's capability.

The model given in Eq. (1) can be generally extended to consider multiple operator's factors. This can be done by extending variable F as vector $\mathbf{F}=[F_1 \ \ F_2 \ \ \cdots \ \ F_r]^T$ to represent r factors, where superscript T represents transpose operator. If we consider only the additive effect of the operator's factors $F_1, F_2, \ldots, F_r$ (no interactions), the extended model can be represented as

$$\delta(\mathbf{F}) = \Delta_M(1 - e^{-\boldsymbol{\beta}^T\mathbf{F}}) \tag{3}$$

Here, parameter vector $\boldsymbol{\beta}=[\beta_1 \ \ \beta_2 \ \ \cdots \ \ \beta_r]^T$ is associated with factors $\beta_1, \beta_2, \ldots, \beta_r$ to account for the relative sensitivity of each corresponding factor on the improved quality conforming rate.

The following discussion will further show how to implement the model by substituting the specific operator's factors of D, L, and C into Eq. (3). For the purpose of developing a general model, these operator's factors will be normalized by their corresponding dimensionless factors, which is discussed in detail below.

2.1.1 Thinking Time. The thinking time, denoted by C, is defined as the time for mental deliberation activities that is needed for executing a specific production task. We refer to *task cycle time* as the length of time needed at a station to complete its production task. Thinking time is used by the operator to perform cognitive activities with the aim of improving his or her quality conforming performance at the station. These cognitive activities include, but are not limited to, deliberating, comparing, remembering, and associating. If we denote the production task cycle time by CT, the minimum time required to complete a physical operation to produce a unit at the station by CT_0, and the mental deliberation thinking time by C, the production task cycle time at a given station CT satisfies $\mathrm{CT}=\mathrm{CT}_0+C$. The normalized cycle time ρ_C is defined as

$$0 \le \rho_C = \frac{\mathrm{CT} - \mathrm{CT}_0}{\mathrm{CT}_M - \mathrm{CT}_0} \le 1 \tag{4}$$

where CT_M corresponds to the maximum value for cycle time, beyond which there is no further significant improvement in the process quality conforming rate even if operators were given more deliberation thinking time than $\mathrm{CT}_M - \mathrm{CT}_0$.

2.1.2 Choice Task Complexity Induced by Product Variety. The demanded product variety increases the mental workload imposed on operators by complicating certain cognitive activities such as recognizing what part type to produce and/or what tool and fixtures to select. It also increases the burdens placed on the operator by requiring him or her to alternate among multiple activities. As proposed by Boer [31], the information entropy H_D will be used to measure the mental workload imposed on an operator due to the choice decision among alternative tasks. Based on Refs. [28,29], if P_i^D is used to denote the mixed ratio that product type i (requiring performing task i) is demanded at a station, the choice task complexity can be represented by

$$H_D = -\sum_i P_i^D \log P_i^D \tag{5}$$

Furthermore, the normalized complexity ρ_D is defined as

$$0 \le \rho_D = \frac{H_D}{H_{D,M}} \le 1 \tag{6}$$

where $H_{D,M}$ is the maximum task complexity represented by the maximum entropy value. This corresponds to a situation in which all product types are demanded with the same frequency, which is represented by the entropy of a random variable with n equally likely outcomes [32].

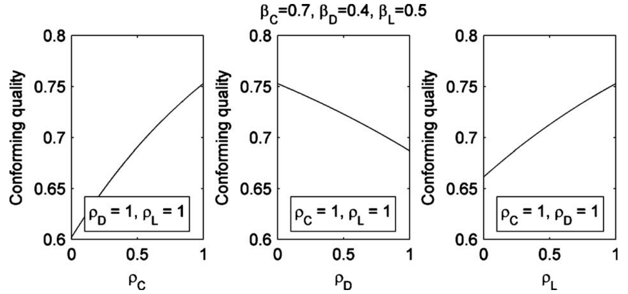

Fig. 2 The effect of three operator's factors on process quality

2.1.3 Operator's Experience. This term refers to the improvement in an operator's performance gained through autonomous learning by the repetitive completion of a task. In this paper, we will measure experience in terms of the number of cumulative units produced by an operator. Similarly, the normalized complexity ρ_L is defined as

$$0 \leq \rho_L = \frac{L}{L_M} \leq 1 \tag{7}$$

where L_M corresponds to the effective learning period. The interpretation of L_M is that there is no further significant improvement in the process quality conforming rate after operators spent a sufficient amount of time executing a task, i.e., when $L > L_M$.

By substituting the above three normalized operator's factors ρ_C, ρ_D, and ρ_L into Eq. (3), the proposed integrated model takes the form of

$$Q(\rho_C,\rho_D,\rho_L) = Q_0[1 + \delta(\rho_C,\rho_D,\rho_L)] \tag{8}$$

$$\delta(\rho_C,\rho_D,\rho_L) = \Delta_M(1 - e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L}) = \Delta_M(1 - e^{-\boldsymbol{\beta}^T\boldsymbol{\rho}}) \tag{9}$$

where vector $\boldsymbol{\rho}=[\rho_C \quad \rho_D \quad \rho_L]^T$ and $\boldsymbol{\beta}=[\beta_C \quad \beta_D \quad \beta_L]^T$. Figure 2 shows the effect trend of each of the three factors (ρ_C, ρ_D, and ρ_L) on the quality conforming rate when the other two factors are fixed.

2.2 Analysis of the Effect of Cycle Time on Production Throughput at a Single Station.

Modeling the throughput traditionally assumes a cycle time-independent quality conforming rate Q. Since our proposed model relates the production CT to the quality conforming rate Q, based on Little's law [33], the throughput under the proposed framework can be expressed as

$$\mathrm{TH} = \frac{Q \times \mathrm{WIP}}{\mathrm{CT}} = \frac{[Q_0 + Q_0\Delta_M(1 - e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L})] \times \mathrm{WIP}}{\rho_C(\mathrm{CT}_M - \mathrm{CT}_0) + \mathrm{CT}_0} \tag{10}$$

where WIP is the work in process defined as the number of units within the manufacturing system. From Eq. (10), we can see that increasing the cycle time factor ρ_C not only increases the denominator CT but also increases the numerator Q. Hence, the overall effect on the throughput should be carefully studied.

For a fixed CT and WIP, the throughput can be trivially maximized at ρ_D=1 and ρ_L=1 by achieving the maximum Q. Since the effect of increasing factor ρ_C on the numerator (Q) is not the same as its effect on the denominator (CT), the resulting effect on TH varies with different ρ_C. Furthermore, since the increment rate of the denominator is $\mathrm{CT}_M-\mathrm{CT}_0$ while the increment rate of the numerator is $\mathrm{WIP}\times Q_0\Delta_M\beta_C e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L}$, it yields that function TH is convex. Proposition 1 below provides a way of finding the optimal ρ_C^* that maximizes TH at a given station.

PROPOSITION 1. *For a given set of fixed parameters* WIP, ρ_D *and* ρ_L, *the maximum throughput* TH^* *occurs at* ρ_C^* *satisfying*

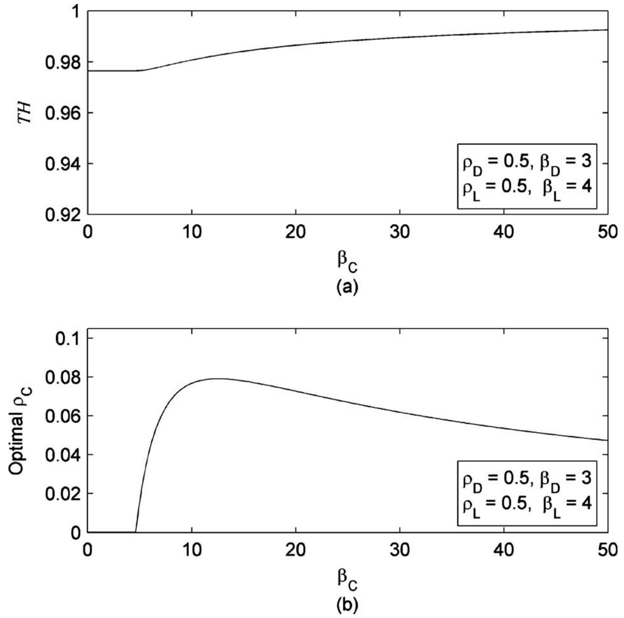

Fig. 3 Maximum throughput as a function of β_C and optimal ρ_C

$$\mathrm{TH}^* = \frac{\mathrm{WIP} \times Q_0\Delta_M\beta_C e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L}}{\mathrm{CT}_M - \mathrm{CT}_0} \tag{11}$$

Proof of Proposition 1. By taking the derivative on TH in Eq. (10) and substituting $\mathrm{TH}^*=\mathrm{WIP}\times Q/\mathrm{CT}$, it yields

$$\begin{aligned}\frac{\partial \mathrm{TH}}{\partial \rho_C} &= -\frac{Q \times \mathrm{WIP}}{\mathrm{CT}^2}(\mathrm{CT}_M - \mathrm{CT}_0) \\ &\quad + \frac{\mathrm{WIP}}{\mathrm{CT}}(Q_0\Delta_M\beta_C e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L}) \\ &= -\frac{1}{\mathrm{CT}}[\mathrm{TH}(\mathrm{CT}_M - \mathrm{CT}_0) - \mathrm{WIP} \\ &\quad \times (Q_0\Delta_M\beta_C e^{-\beta_C\rho_C-\beta_D\rho_D-\beta_L\rho_L})]\end{aligned}$$

By setting $\partial\mathrm{TH}/\partial\rho_C=0$, we obtain Eq. (11). Thus, Proposition 1 is proven. ■

A useful application of Eq. (11) is to find the maximum throughput achieved under different levels of factor ρ_C. Figure 3(*a*) shows the throughput as a function of parameter β_C while the other two parameters are fixed. Figure 3(*b*) corresponds to the optimal value of ρ_C, under which β_C maximizes the throughput.

An interesting conclusion can be drawn from Fig. 3. When the small value of β_C ($\beta_C \leq 5$) is used, it results in optimal ρ_C=0, i.e., there is no deliberation thinking time allocated. This is because when β_C is small, the gain in the conforming quality by allowing the deliberation time is not competitive compared with the loss of production rate due to the increase of the cycle time.

2.3 Analysis of Throughput at an Assembly Line.

When considering two or more stations in an assembly system, allocating process cycle times at each station constitutes an important issue in assembly line design; this is known as the "line balancing problem" [34,35]. The main objective of line balancing is to maximize the throughput of the system by reducing the difference of cycle times at individual stations. Since we assume that the task cycle time at each station has an effect on its quality conforming rate, the line balancing problem acquires a new dimension due to the fact that the output quality conforming rate and throughput are both affected by the cycle time assigned to each station. Moreover, the entropy of the input mixed ratio transferred from upstream stations to downstream stations increases as the percentage of nonconforming units increases. This interaction effect can be analytically studied by considering the effect of β_i^k, which corresponds to factor i $(i=C,D,L)$ at station k.

In this section, we discuss these interrelated effects in the context of an assembly system with stations in a serial configuration. At this point, it should be clarified that the main assumption throughout this section is that there is a fixed total production cycle time $\text{CT}^{E(1,2,\ldots,n)}$ corresponding to the cycle time of the system formed by station $1,2,\ldots,n$ as

$$\text{CT}^{E(1,2,\ldots,n)} = \text{CT}^1 + \text{CT}^2 + \cdots + \text{CT}^n \tag{12}$$

where CT^k is the cycle time at station k $(k=1,2,\ldots,n)$. In other words, the total production cycle time corresponds to the time that elapses from the moment when a unit enters the line as raw material to the instant that it leaves the line as a finished product. The time spent on activities that are not directly relevant to production, such as transportation from one station to another, and time spent waiting in buffers are not considered in this paper.

For an assembly line having n stations denoted by $S_{E(1,2,\ldots,n)}$, the overall process quality conforming rate is defined as $Q^{E(1,2,\ldots,n)}=Q^1Q^2\cdots Q^n$. Here, Q^k is the individual quality conforming rate at station k, which depends on the individual cycle time CT^k at station k, based on Eq. (8). Therefore, the analysis of the effect on process quality and throughput due to reducing individual CT^k at one or more stations needs to be carefully studied. This is discussed in detail below.

Suppose that Q^1 and Q^2 are used to characterize the quality conforming rate at two stations, stations 1 and 2, respectively, as follows:

$$Q^1 = Q_0^1[\Delta_M^1(1 - e^{-\beta_C^1\rho_C^1-\beta_D^1\rho_D^1-\beta_L^1\rho_L^1})] \quad \text{and}$$
$$Q^2 = Q_0^2[\Delta_M^2(1 - e^{-\beta_C^2\rho_C^2-\beta_D^2\rho_D^2-\beta_L^2\rho_L^2})]$$

where Q_0^k and Δ_M^k are the minimal quality conforming rate and maximum possible percentage of quality improvement at station k $(k=1,2)$, respectively. Furthermore, factor ρ_i^k corresponds to the normalized operator's factor ρ_i at station k for $i=C,D,L$.

(i) *Station 1.* An increase of CT^1 by Δ_C^1 will increase ρ_C^1 to

$$\rho_{C+\Delta_C}^1 = \rho_C^1 + \frac{\Delta_C^1}{\text{CT}_M^1 - \text{CT}_0^1} \tag{13}$$

leading to an increase in Q^1 of ΔQ^1 given by

$$\Delta Q^1 = K^1 Q_0^1 \Delta_0^1 (1 - e^{\beta_C^1\delta_C^1}) \tag{14}$$

where $K^1=e^{-\beta_C^1\rho_C^1-\beta_D^1\rho_D^1-\beta_L^1\rho_L^1}$ and $\delta_C^1=\Delta_C^1/(\text{CT}_M^1-\text{CT}_0^1)$.

(ii) *Station 2.* If CT^1+CT^2 is fixed, an increment of CT^1 to $\text{CT}^1+\Delta_C^1$ will decrease CT^2 to $\text{CT}^2-\Delta_C^1$, which consequently decreases ρ_C^2 as

$$\rho_{C+\Delta_C}^2 = \rho_C^2 - \frac{\Delta_C^1}{\text{CT}_M^2 - \text{CT}_0^2} \tag{15}$$

The analysis of the effect of increasing CT^1 to $\text{CT}^1+\Delta_C^1$ on Q^2 demands more consideration since an increase of Q^1 by ΔQ^1 (Eq. (14)) also affects the propagated input entropy H_D^2 at station 2. This is because there is a proportion $1-Q^1$ of nonconforming units entering to station 2, which leads to further inspection efforts. This increases H_D^2 and thus complicates the process. Mathematically, the input mixed ratio entering to station 2 can be represented by $p_1^2=Q^1p_1^1$ and $p_2^2=Q^1p_2^1$, where p_i^k is the input mixed ratio for task i and station k. Thus, $p_\varepsilon^2=1-Q^1$, where p_ε^2 corresponds to the nonconforming product rate entering to station 2. Furthermore, the input complexity at station 2 can be calculated by

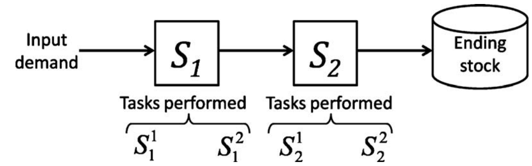

Fig. 4 Two station manual assembly process

$$H_D^2 = Q^1 H_D^1 + H_{Q^1} \tag{16}$$

where $H_{Q^1}=-Q^1 \log Q^1-(1-Q^1)\log(1-Q^1)$. Hence, an increase of ΔQ^1 at station 1 decreases the input entropy $H_D^2=Q^1H_D^1+H_{Q^1}$ by

$$\Delta_H^2 = H_D^1 \Delta Q^1 + H_{Q^1+\Delta Q^1} - H_{Q^1} \tag{17}$$

Overall, the decrease on Q^2 due to increasing CT^1 by Δ_C^1 can be computed as

$$\Delta Q^2 = K^2 Q_0^2 \Delta_0^2 (e^{\beta_C^2\delta_C^2-\beta_D^2\delta_D^2} - 1) \tag{18}$$

where $K^2=e^{-\beta_C^2\rho_C^2-\beta_D^2\rho_D^2-\beta_L^2\rho_L^2}$, $\delta_C^2=\Delta_C^1/(\text{CT}_M^2-\text{CT}_0^2)$, and $\delta_D^2 =(H_D^1\Delta Q^1+H_{Q^1+\Delta Q^1}-H_{Q^1})/H_M$.

Finally, the effect on the overall process quality $Q^{E(1,2)}=Q^1Q^2$ after increasing the cycle time of station 1 from CT^1 to $\text{CT}^1 +\Delta_C^1$ is

$$\Delta Q^{E(1,2)} = \Delta Q^1 Q^2 - \Delta Q^1 \Delta Q^2 - Q^1 \Delta Q^2 \tag{19}$$

By using Eq. (19), we can maximize $Q^{E(1,2)}$ or throughput by finding the optimum CT^1, thus finding the optimum CT^2.

3 System Performance Analysis

In this section, we will present two examples to illustrate the use of the proposed model. The first example illustrates how the process quality conforming rate and throughput are affected by allocating the production cycle times in a simple assembly line consisting of two serial stations. The second example further illustrates how to allocate the cycle time at individual stations to maximize the process capability for handling complexity in an assembly line with five stations.

3.1 Example 1: Two Stations in a Serial Configuration

3.1.1 Analysis of Quality Conforming Rate. We consider a simplified assembly process consisting of two sequential stations requiring intensive manual operations, as shown in Fig. 4. S_k^i corresponds to task i at station k for producing a product of type k. The process parameters at each station are given in Table 1. Table 2 shows the input demand mixed ratio at each station corresponding to each product variety.

Based on Eq. (5), the entropy of the demand variety at station 1 is obtained as H_D^1=0.5623. As discussed in Sec. 2.3, the quality conforming rate at station 1, denoted as Q^1, affects the entropy of the input variety at station 2, which leads to the increase of H_D^2 and thus complicates the process. The input complexity at station 2 can be calculated by Eq. (16).

Assume that the combined total production cycle time of these two stations is constrained to being less than 200 s. A production engineer is required to split the total production cycle time into the individual cycle times of each station in order to achieve a desired optimal process performance. In other words, given $\text{CT}^{E(1,2)}=\text{CT}^1+\text{CT}^2$, the engineer must determine how to maximize the quality conforming rate or the throughput by determining CT^1. Figures 5(*a*)–5(*c*) show the effect of varying CT^1 on Q^1, Q^2, and $Q^{E(1,2)}$, respectively. Note that $\text{CT}^{E(1,2)}=\text{CT}^1+\text{CT}^2=200$ s. The analysis results are also reported in Table 3.

We can see in Fig. 5(*c*) that the maximum quality conforming rate for the production line is $Q^{E(1,2)^*}$=0.79, which is achieved

Table 1 Process parameters for the proposed model

Station k	Tasks at station k	$CT^k_{i,0}$	$CT^k_{i,M}$	$Q^k_{i,0}$	$\Delta^k_{i,M}$
Station 1	S^1_1 and S^2_1	$CT^1_{1,0}=CT^1_{2,0}=30$	$CT^1_{1,M}=CT^1_{2,M}=190$	$Q^1_{1,0}=Q^1_{2,0}=0.8$	$\Delta^1_{1,M}=\Delta^1_{2,M}=0.25$
Station 2	S^1_2 and S^2_2	$CT^2_{1,0}=CT^2_{2,0}=70$	$CT^2_{1,M}=CT^2_{2,M}=95$	$Q^2_{1,0}=Q^2_{2,0}=0.8$	$\Delta^2_{1,M}=\Delta^2_{2,M}=0.25$
		Station 1		Station 2	
	β^1_C	0.70568	β^2_C	1.0836	
	β^1_D	0.37084	β^2_D	0.7889	
	β^1_L	0.48344	β^2_L	0.9194	

when station 1's cycle time is allocated as $CT^{1^*}=105$; thus, the optimal cycle time for station 2 is $CT^{2^*}=CT^{E(1,2)}-CT^{1^*}=200-105=95$.

3.1.2 Analysis of Throughput. Suppose that the goal is to maximize the effective throughput of the whole assembly line. For the system shown in Fig. 4, the capacity of the bottleneck station corresponds to $r_b=\min(TH^1, TH^2)$, where TH^k denotes the throughput of station k. From Little's law [33], we know that under the assumption of deterministic cycle times, the WIP that achieves maximum throughput is $WIP=Q^{E(1,2)}r_b CT^{E(1,2)}=0.7900\times 0.0081\times 200=1.2798$.

Figure 6 illustrates the effects of varying CT^1 on $TH^{E(1,2)}$. The maximum $TH^{E(1,2)^*}$ by considering $Q^{E(1,2)}$ occurs at $CT^1=93.84$, while the maximum $TH^{E(1,2)^*}$ without considering $Q^{E(1,2)}$ occurs at $CT^1=100$. As we can see, there is an overly optimistic prediction of $TH^{E(1,2)^*}$ when the study fails to consider the operator's effect on $Q^{E(1,2)}$.

Table 2 Input mixed ratio for two station scenario

	Mixed ratio task 1	Mixed ratio task 2
Station 1	$\Pr(S^1_1)=0.75$	$\Pr(S^2_1)=0.25$
Station 2	$\Pr(S^1_2)=0.40$	$\Pr(S^2_2)=0.60$

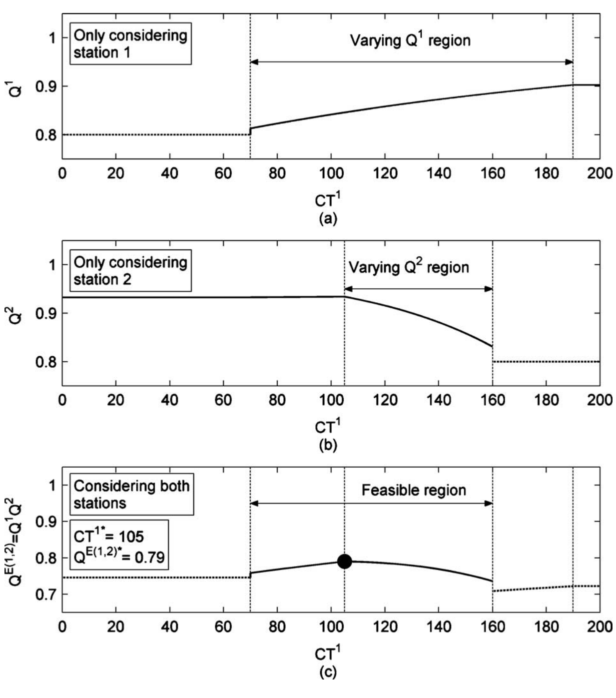

Fig. 5 CT^1 versus (*a*) Q^1, (*b*) Q^2, and (*c*) $Q^{E(1,2)}$

3.2 Example 2: Five Stations With a Hybrid Configuration. The allocation of cycle times for individual stations is a critical issue in the design of a manufacturing system, especially for MMASs with different product types. In the example shown in Fig. 7, an assembly system consisting of five stations producing three different product types is used to produce a table. We assume that every product must pass through all stations.

The objective is to maximize the PCC, which is defined in Ref. [36], by optimally allocating individual station's cycle time. Mathematically, the problem is formulated as

$$\max_{CT^k} \text{PCC}$$

such that

$$CT^{E(1,2,3,4,5)} \le 200 \text{ s}$$

$$CT^k_i \ge CT^k_{i,0} \quad \text{for } i=1,2,3 \quad \text{and} \quad k=1,2,3,4,5$$

Based on Ref. [37], PCC is calculated by

$$\text{PCC} = \sum_{i,j} \pi^{\text{in,out}}_{ij} \log \frac{\pi^{\text{in,out}}_{ij}}{\pi^{\text{in}}_i \pi^{\text{out}}_j} \tag{20}$$

where π^{in}_i is the percentage of the input demand corresponding to the ith product type and π^{out}_j corresponds to the percentage of products of type j produced including $\pi^{\text{out}}_\varepsilon$, which is the percentage of nonconforming products produced by the system. Element $\pi^{\text{in,out}}_{ij}$ is the percentage of demanding a product type i and producing a product type j. The relation $\pi^{\text{in,out}}_{ij}=\psi^{E(\cdot)}_{ij}\pi^{\text{in}}_i$ is held, where $\psi^{E(1,2,3,4,5)}_{ij}$ is the entry at the ith row and jth column of matrix $\mathbf{\Psi}^{E(1,2,3,4,5)}$ given by

Table 3 Analysis results for two station scenario

	Independent optimal conforming rate Q^{k^*}	Combined optimal conforming rate Q^{k^*} to achieve $Q^{E(1,2)^*}$	Optimal process time CT^{k^*}	Throughput at individual stations TH^k
Station 1	0.9025	0.8458	105	0.0081
Station 2	0.9340	0.9340	95	0.0098
System	-	0.7900	200	-

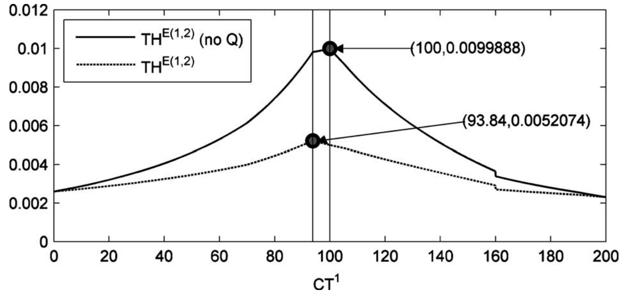

Fig. 6 The effect of varying CT^1 on throughput

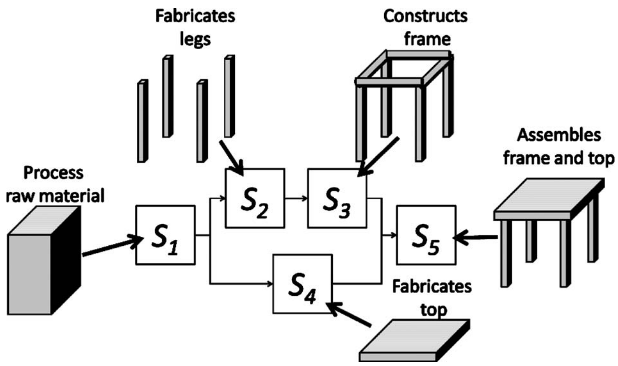

Fig. 7 Production process for assembling a table

$$\boldsymbol{\Psi}^{E(1,2,3,4,5)} = \begin{bmatrix} \psi_{00}^{E(1,2,3,4,5)} & 0 & \cdots & 0 & \psi_{0,\varepsilon}^{E(1,2,3,4,5)} \\ 0 & \psi_{11}^{E(1,2,3,4,5)} & \cdots & 0 & \psi_{1,\varepsilon}^{E(1,2,3,4,5)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \psi_{N-1,N-1}^{E(1,2,3,4,5)} & \psi_{N-1,\varepsilon}^{E(1,2,3,4,5)} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{21}$$

where $\psi_{11}^{E(1,2,3,4,5)}=\Pr\{$"producing a conforming product type i at the assembly system"$\}$ and $\psi_{1\varepsilon}^{E(1,2,3,4,5)}=\Pr\{$"producing a defective product type i at the assembly system"$\}$. Furthermore, based on the model proposed in this paper, the element ψ_{ii}^{k} is obtained by

$$\psi_{ii}^{k} = Q_{i,0}^{k}[1+\Delta_{i,M}^{k}(1-e^{-\beta_{i,C}^{k}\rho_{i,C}^{k}})] \tag{22}$$

where $Q_{i,0}^{k}$ is the minimal quality conforming rate at station k for product type i. Similarly, $\Delta_{i,M}^{k}$ is the upper limit of the possible improvement of the quality conforming rate at station k for product type i when the operator's factors are under the most favorable conditions. Furthermore, β_{C}^{k} and ρ_{C}^{k} are the cycle time parameter and the normalized cycle time factor, respectively, at station k. For simplicity, we have considered ρ_{C}^{k} as the only operator's factor affecting the quality conforming performance since the other factors (ρ_{D}^{k} and ρ_{L}^{k}) do not affect our optimization solution. The parameters for obtaining ψ_{ii}^{k} are given in Table 4 where $\mathrm{CT}_{i,M}^{k}$ is the maximum cycle time at station k for product type i, after which there is no further quality conforming improvement.

To obtain the overall process quality conforming matrix $\boldsymbol{\Psi}^{E(1,2,3,4,5)}$ from the individual quality conforming matrix $\boldsymbol{\Psi}^{k}$ at station k, the following algebraic quality expression is obtained based on the method proposed in Ref. [37]:

$$\boldsymbol{\Psi}^{E(1,2,3,4,5)} = \boldsymbol{\Psi}^{1}\otimes_{Q}[(\boldsymbol{\Psi}^{2}\otimes_{Q}\boldsymbol{\Psi}^{3})\oplus_{Q}\boldsymbol{\Psi}^{4}]\otimes_{Q}\boldsymbol{\Psi}^{5} \tag{23}$$

Table 4 Model parameters for producing product type 1, product type 2, and product type 3

Product type 1

	Parameters of proposed model			Processing times (sec)	
Station k	$Q_{1,0}^{k}$	$\Delta_{1,M}^{k}$	$\beta_{1,C}^{k}$	$\mathrm{CT}_{1,0}^{k}$	$\mathrm{CT}_{1,M}^{k}$
Station 1	0.95	0.052	0.80	15	150
Station 2	0.97	0.030	0.70	25	100
Station 3	0.94	0.063	0.60	20	90
Station 4	0.95	0.052	0.80	15	100
Station 5	0.97	0.030	0.90	12	70

Product type 2

	Parameters of proposed model			Processing times (sec)	
Station k	$Q_{2,0}^{k}$	$\Delta_{2,M}^{k}$	$\beta_{2,C}^{k}$	$\mathrm{CT}_{2,0}^{k}$	$\mathrm{CT}_{2,M}^{k}$
Station 1	0.85	0.176	0.90	20	120
Station 2	0.95	0.052	0.85	10	100
Station 3	0.97	0.030	0.85	5	120
Station 4	0.92	0.086	0.70	7	110
Station 5	0.98	0.020	0.65	10	90

Product type 3

	Parameters of proposed model			Processing times (sec)	
Station k	$Q_{3,0}^{k}$	$\Delta_{3,M}^{k}$	$\beta_{3,C}^{k}$	$\mathrm{CT}_{3,0}^{k}$	$\mathrm{CT}_{3,M}^{k}$
Station 1	0.89	0.123	0.70	10	90
Station 2	0.98	0.020	0.75	15	200
Station 3	0.94	0.063	0.85	10	180
Station 4	0.95	0.052	0.80	15	150
Station 5	0.97	0.030	0.80	17	140

Table 5 Weights for generating new chromosomes in each population

	Percentage (%)
Previous population	5
Mutation	20
Crossover	25
Newly generated	50

We assume that the total process cycle time $\mathrm{CT}^{E(1,2,3,4,5)}$ cannot exceed 200 s where $\mathrm{CT}^{E(1,2,3,4,5)}$ can also be obtained based on the following algebraic cycle time expression:

$$\mathrm{CT}^{E(1,2,3,4,5)} = \mathbf{CT}^1 \otimes_Q [(\mathbf{CT}^2 \otimes_Q \mathbf{CT}^3) \oplus_Q \mathbf{CT}^4] \otimes_Q \mathbf{CT}^5 \quad (24)$$

In Eq. (24), $\mathbf{CT}^k = [\mathrm{CT}_1^k \quad \mathrm{CT}_2^k \quad \mathrm{CT}_3^k]$ is the cycle time vector at station k ($k=1,2,3,4,5$), where the element CT_i^k is the cycle time for producing product type i at station k. For simplicity, we will assume that $\mathrm{CT}_i^k = \mathrm{CT}_j^k$ for $k=1,2,3,4,5$. Furthermore, $\mathrm{CT}_{i,0}^k$ is the minimum cycle time required at station k to produce one product of type i.

We propose to use a genetic algorithm (GA) to solve this optimization problem because of its complicated structure. The main concepts of GA are defined as follows.

i. *Chromosomes.* The chromosomes are the potential solutions to the problem. In this example, the chromosomes are five-element vectors of the form

$$\mathbf{CH} = [\mathrm{CT}^1 \quad \mathrm{CT}^2 \quad \mathrm{CT}^3 \quad \mathrm{CT}^4 \quad \mathrm{CT}^5]^T \quad (25)$$

satisfying $\mathrm{CT}^{E(1,2,3,4,5)} \le 200$ s and $\mathrm{CT}_i^k \ge \mathrm{CT}_{i,0}^k$ for $i=1,2,3$ and $k=1,2,3,4,5$. Here, for simplicity, we set $\mathrm{CT}^k = \mathrm{CT}_i^k$, i.e., the same cycle time is assigned for producing each product type i at station k.

ii. *Population.* The set of all chromosomes at each step of the GA.

iii. *Crossover.* Crossover is the act of combining two chromosomes, thus producing a new chromosome with characteristics similar to both of its parents. In this example, the crossover between chromosomes $\mathbf{CH}_i$ and $\mathbf{CH}_j$ is obtained by

$$\mathbf{CH}_c = [(1+\lambda)\mathbf{CH}_i + (1-\lambda)\mathbf{CH}_i]/2 \quad (26)$$

for $-1 \le \lambda \le 1$. In this example, we consider $\lambda = 0.5$.

iv. *Mutation.* When a chromosome mutates, it undergoes a small change, giving rise to new chromosomes. Here, the mutation of chromosome $\mathbf{CH}$ to chromosome $\mathbf{CH}_m$ is defined as

Table 6 Parameters and solutions from GA

	Values
Number of runs of GA	1000
Size of each population	100
Mean solution	1.3038
Standard deviation of solutions	0.0003

$$\mathbf{CH}_m = [CH_m^1 \quad CH_m^2 \quad CH_m^3 \quad CH_m^4 \quad CH_m^5]^T \quad (27)$$

where $CH_m^k = (\mathrm{CT}^k - \varepsilon/5) + \varepsilon\rho_k$ for $k=1,2,3,4,5$ and ε is a positive quantity. The quantity ρ_k is a random variable satisfying $0 \le \rho_k \le 1$ and $\Sigma_k \rho_k = 1$ for $k=1,2,3,4,5$. Here, each ρ_k is equally distributed and $\varepsilon = 30$.

In this example, the number of chromosomes at each population is equal to 100. Four methods are used for obtaining the chromosome at each population: chromosomes from the previous population with high performance, chromosomes obtained from mutation, chromosomes obtained from crossovers, and new chromosomes. Table 5 shows the percentages for each of these four methods used to generate new chromosomes in each population.

Figure 8(*a*) shows the variability of the PCC within the initial population. Figure 8(*b*) shows the optimal PCC attained during the first 100 populations. Table 6 shows the details of the parameters used in the GA as well as statistical characteristics of the solutions obtained through 1000 runs of the GA.

Finally, the average solution among 1000 runs of the GA is

$$\bar{\mathbf{C}}\bar{\mathbf{H}}^* = [77.12 \quad 27.60 \quad 25.36 \quad 50.80 \quad 19.10]^T$$

which provides the optimal cycle time allocation for individual stations under the objective of maximizing the PCC of the system. Furthermore, the standard deviation vector of the solutions is given by

$$\boldsymbol{\sigma}_{\mathbf{CH}^*} = [3.3414 \quad 2.5246 \quad 2.5996 \quad 0.9524 \quad 1.7541]^T$$

Note that the obtained standard deviations are relatively small compared with the corresponding mean values, and, thus, the obtained solutions converge well.

4 Conclusions and Future Work

This paper develops a general framework to model the effect of operator's factors on the process quality conforming rate and throughput in a mixed model assembly system. The proposed model considers both intrinsic factors (e.g., the operator's mental thinking time and working experience) and extrinsic factors (e.g., product variety induced complexity) based on findings in the cognitive literature. We further use two examples to obtain insights

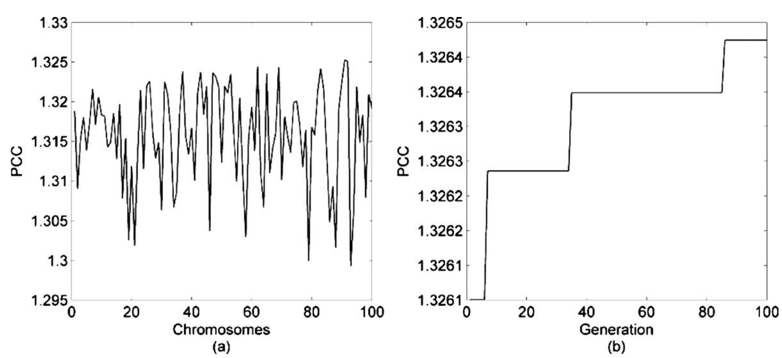

Fig. 8 GA performance

and implications from the proposed model. The examples describe how to allocate each individual station's cycle time in order to achieve the desired optimal process performance including process quality, throughput, and process capability for handling complexity. Future research is needed for conducting further empirical validations of the proposed model through real-world scenarios or experimental tests.

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